

LINEAR INEQUALITIES

The Math of Possibilities.



A Shift in Thinking: Exact vs. Possible

The Exact Point

Ravi has exactly ₹120. A packet of rice is ₹30.

$$30x = 120$$

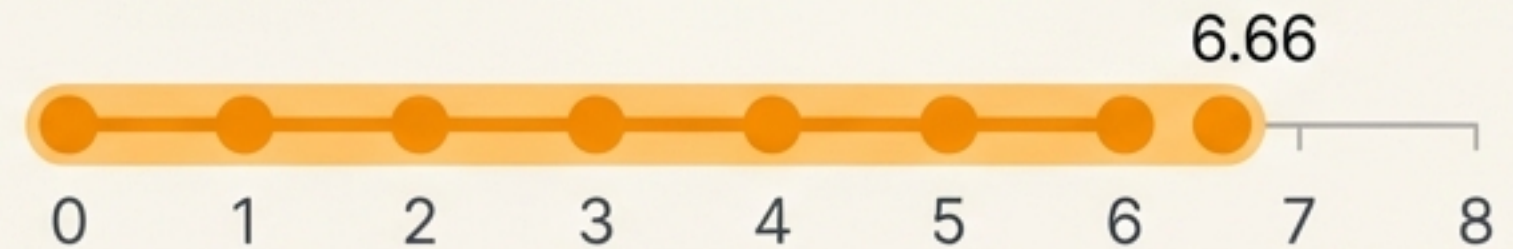


Only one exact answer: $x = 4$

The Continuous Spectrum

Ravi has up to ₹200. A packet of rice is ₹30.

$$30x < 200$$



A range of valid answers: $x \in \{0, 1, 2, 3, 4, 5, 6\}$

Key Insight: Equations ask 'What is the answer?' **Inequalities** ask 'What are all the possible answers?'

The Taxonomy of Inequalities

Numerical (Numbers Only)

Literal (Includes Variables)

Strict Inequalities

No overlapping with the boundary.



$3 < 5$



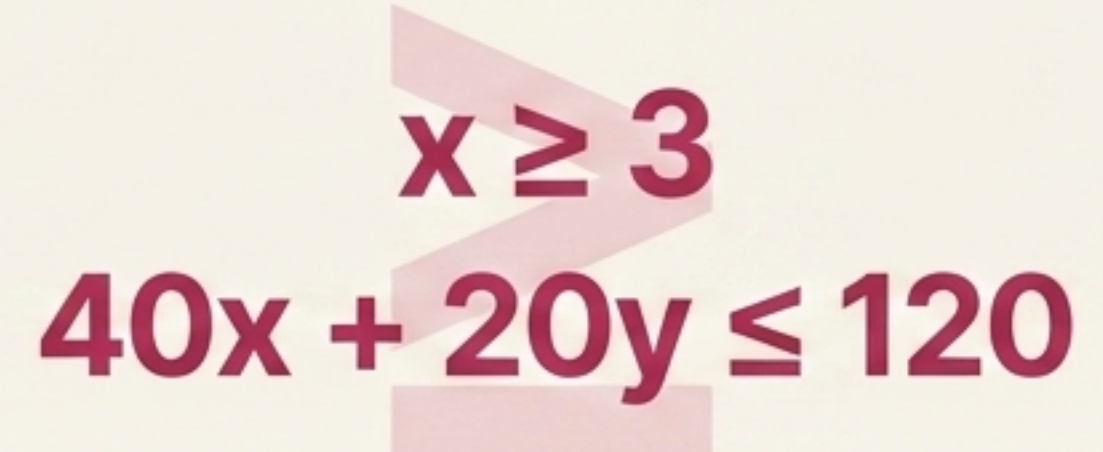
$x > 2$

Slack Inequalities

Includes the boundary point.



$3 \leq 5$



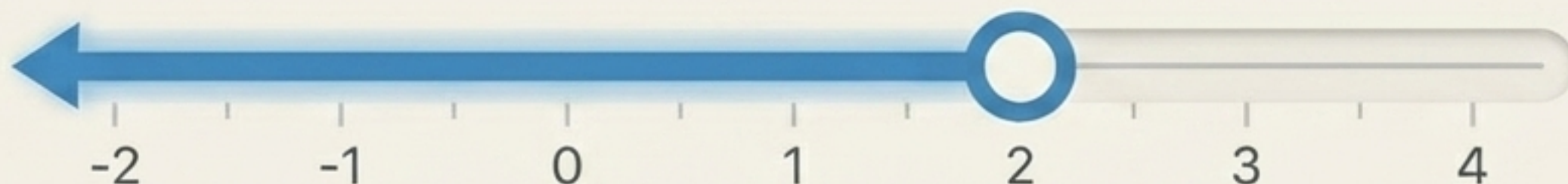
$x \geq 3$
 $40x + 20y \leq 120$

Double Inequalities: Bounding a spectrum from both sides (e.g., $3 < x < 5$).

Mapping the Spectrum

Translating mathematical syntax into visual boundaries.

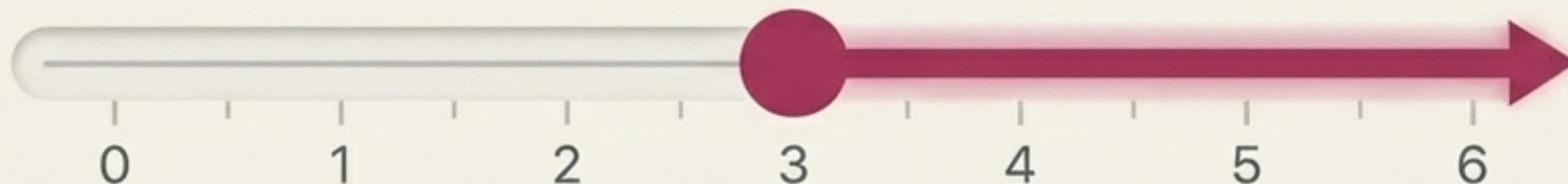
$$x < 2$$



$$(-\infty, 2)$$

Open Circle: The boundary is an invisible wall.
We get infinitely close, but never touch.

$$x \geq 3$$



$$[3, \infty)$$

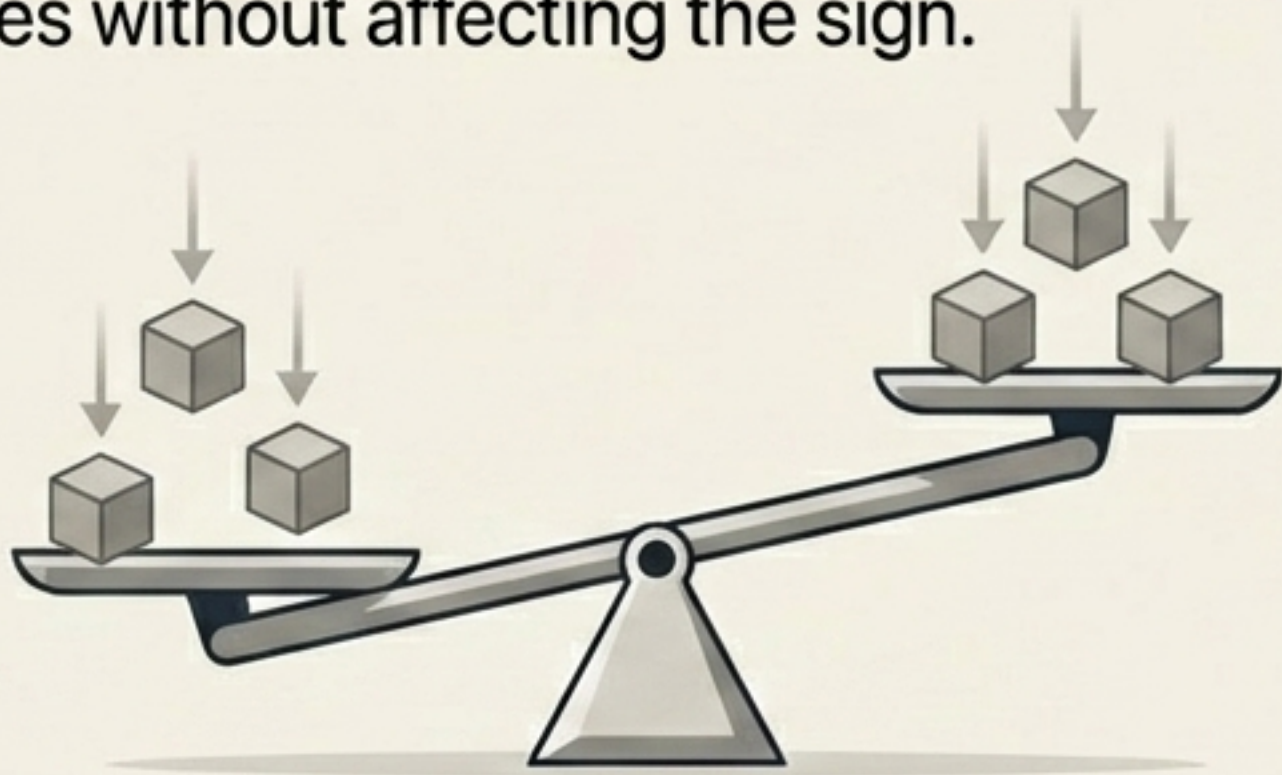
Closed Circle: The boundary is solid ground. We stand precisely on it.

The Golden Rules of Algebra

Standard algebraic balancing applies—with one major exception.

Rule 1: Addition & Subtraction

Add or subtract equal numbers from both sides without affecting the sign.

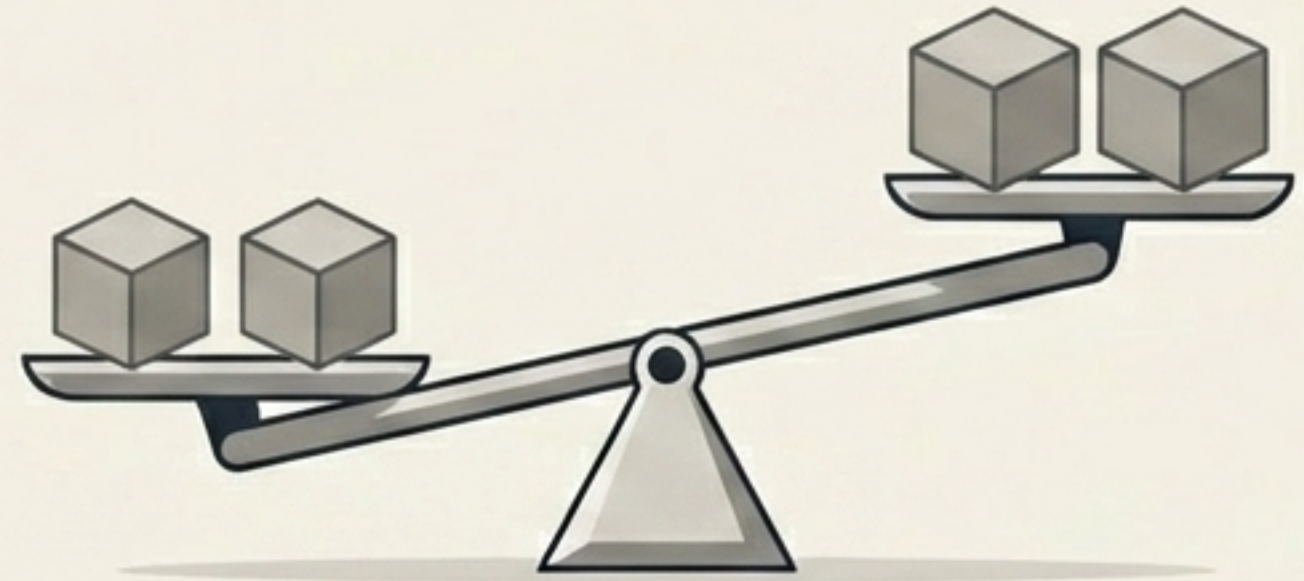


Math Translation:

If $x > 5$, then $(x + 2) > (5 + 2)$.

Rule 2: Positive Multiplication & Division

Multiply or divide both sides by the same positive number.



Math Translation:

If $2x < 4$, then $(2x \div 2) < (4 \div 2)$.

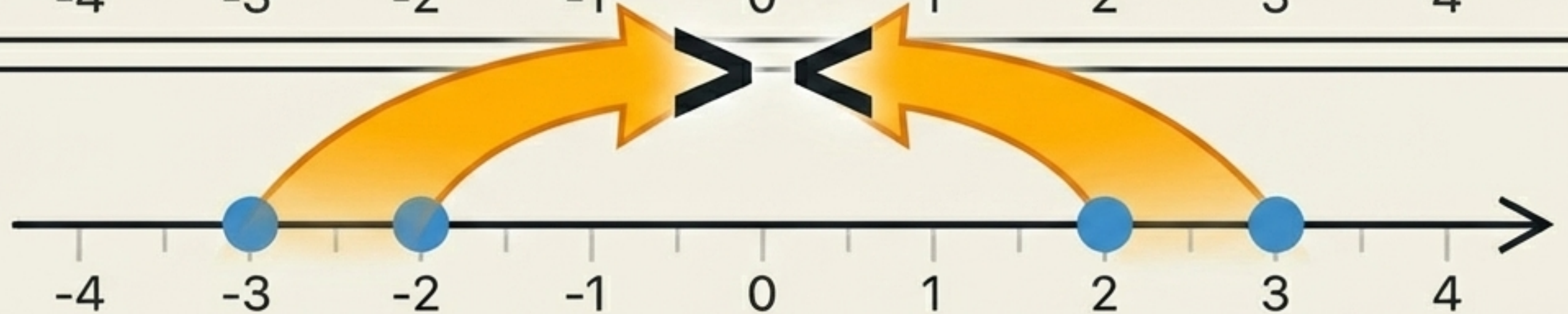
The Danger Rule: The Negative Pivot

When multiplying or dividing by a negative number, the sign **MUST** reverse.

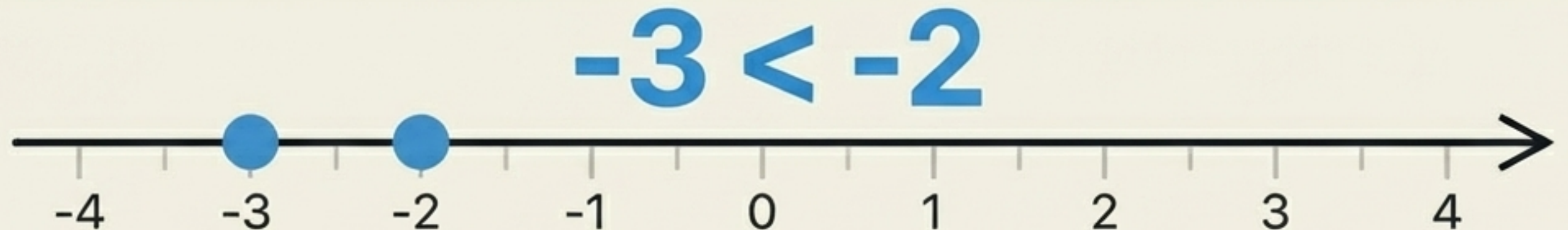
The Initial State



The Action:
Multiply by -1

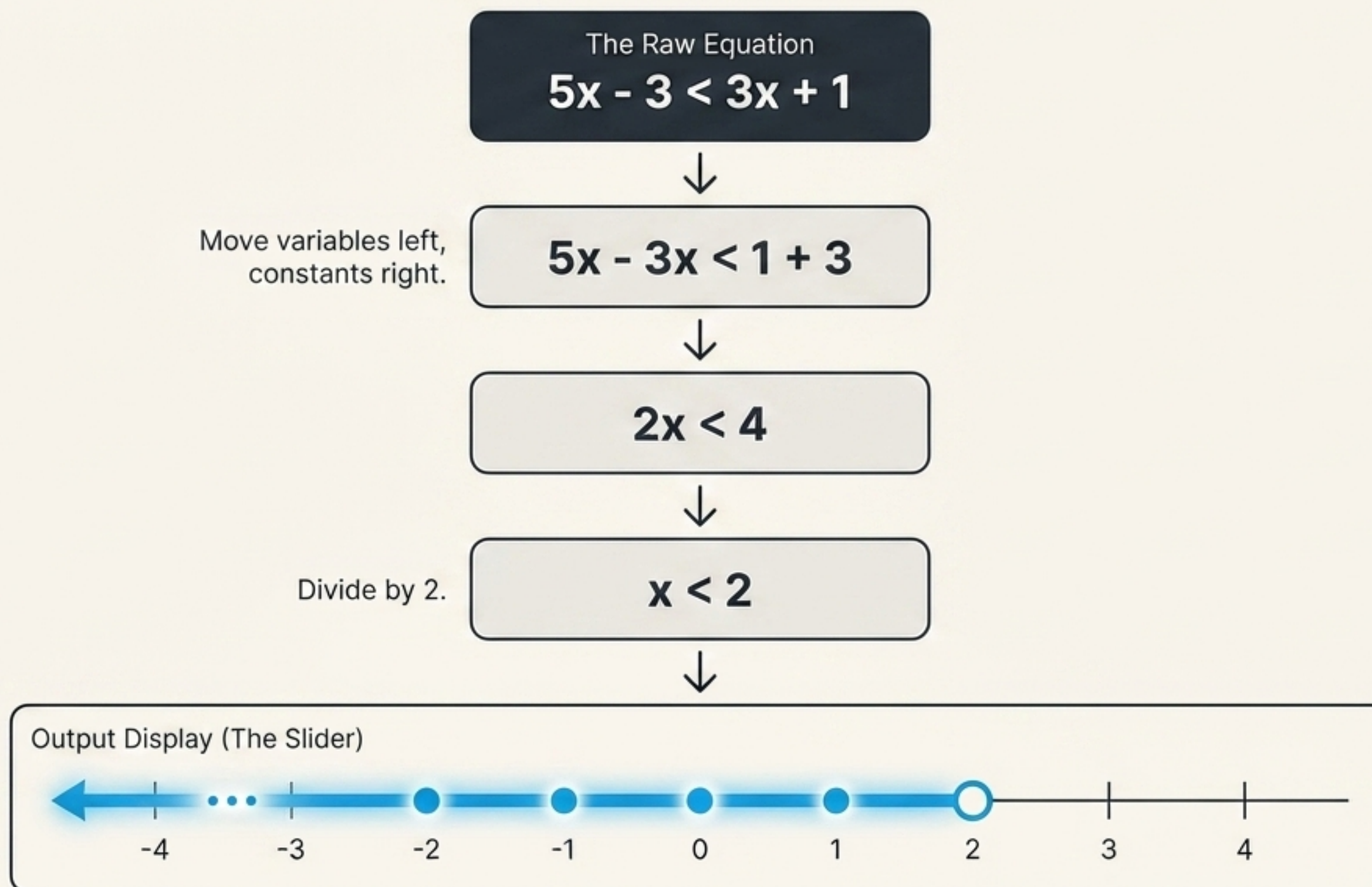


The New
Reality



The Solution Pipeline

Filtering a complex equation into a clean spectrum.

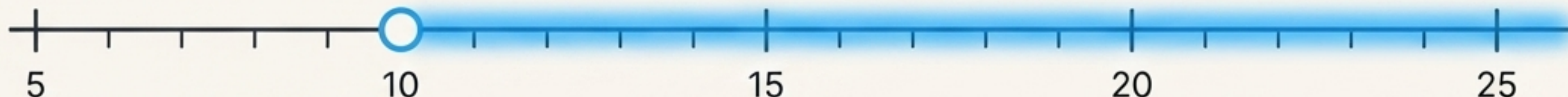


Double Inequalities: Bounding the Spectrum

Find consecutive odd integers (x and $x+2$) larger than 10 whose sum is less than 40.

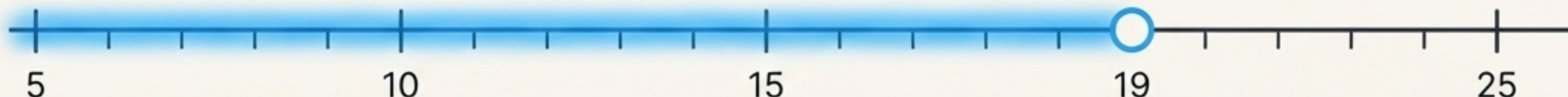
$$x > 10$$

Constraint A



$$2x + 2 < 40 \rightarrow x < 19$$

Constraint B



$$10 < x < 19$$

The Valid Zone



Only the odd integers inside the overlapping spectrum satisfy all constraints.

Real-World Translation Engine

Raw English

A manufacturer has 600L of 12% acid.

Adding x liters of 30% acid.

Goal: Resulting mixture is between 15% and 18%.

Mathematical Syntax

15% < Mixture

Mixture < 18%

$$15\%(x + 600) < 30\%x + 12\%(600) < 18\%(x + 600)$$

Output Display



Safe Zone: 120 Liters to 300 Liters

Inequalities don't just solve math problems; they engineer precise safety margins for real-world chemistry.

The Operator's Manual

Linear Inequalities Visual Cheat Sheet

The Syntax Key

$<, >$

Strict = Open Circle
= Soft Bracket ()

$\leq, \geq$

Slack
= Closed Circle ●
= Hard Bracket []

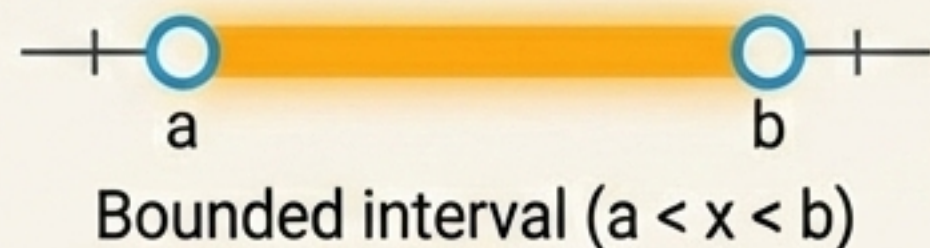
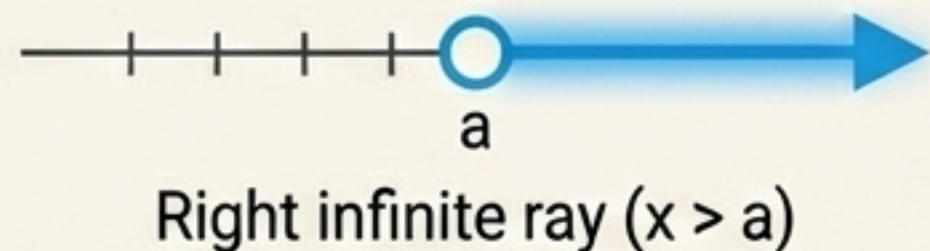
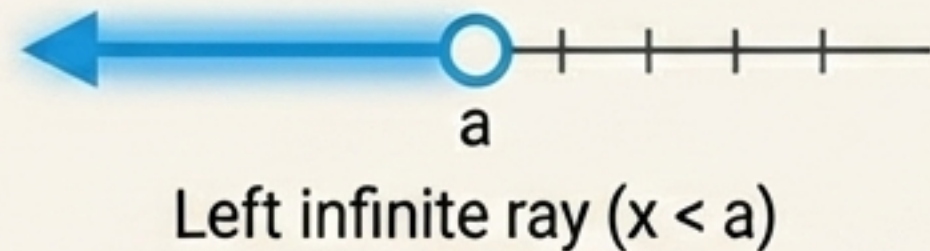
The Golden Rules

$+/ -$ = Safe (No flip)

$\times / \div$ = Safe (No flip)
(Positive)

$\times / \div$ = **DANGER**
(Negative) (Rotate the sign 180°)

The Spectrum Mappings



"Mathematics is the art of saying many things in many different ways." – Maxwell