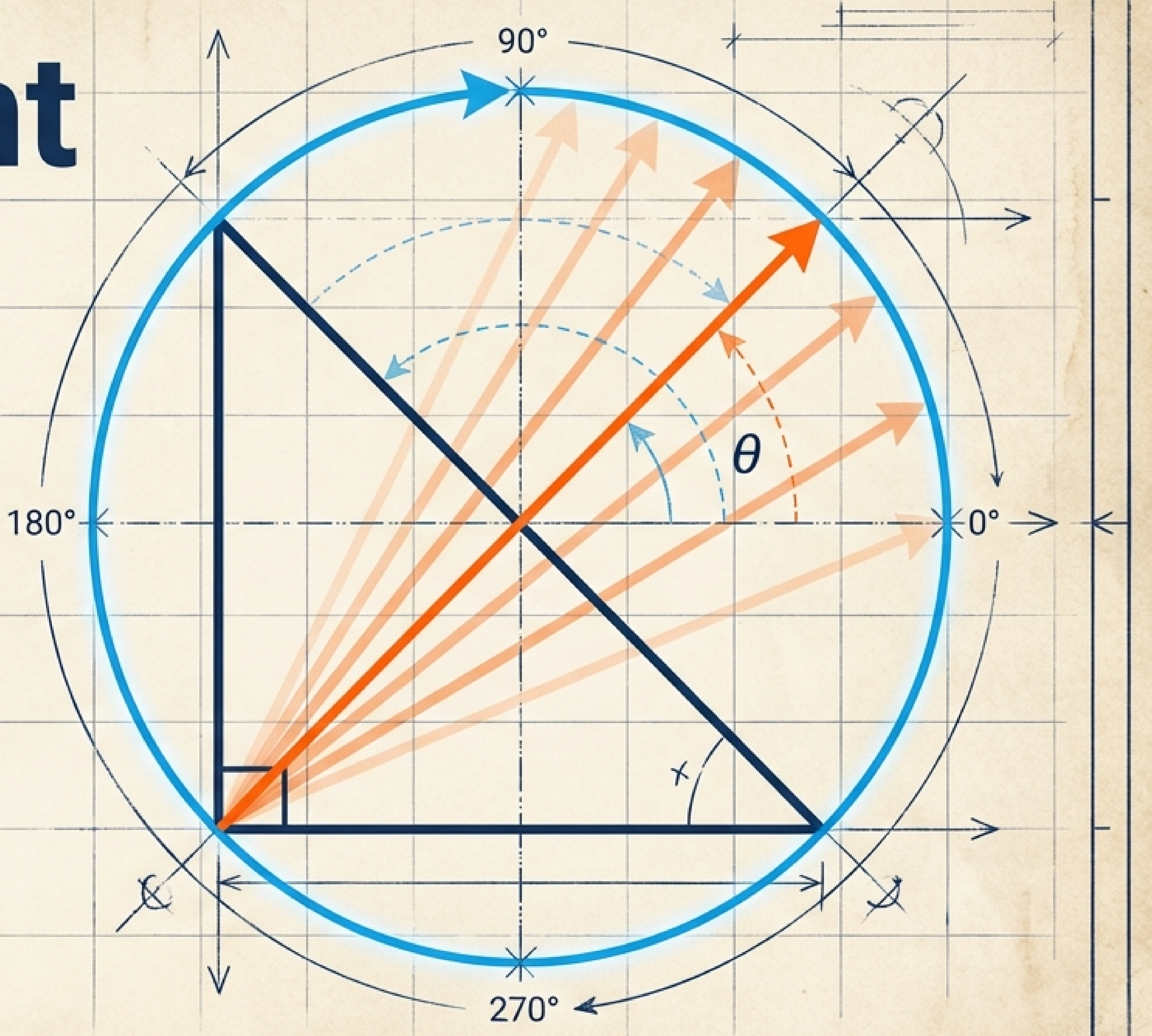
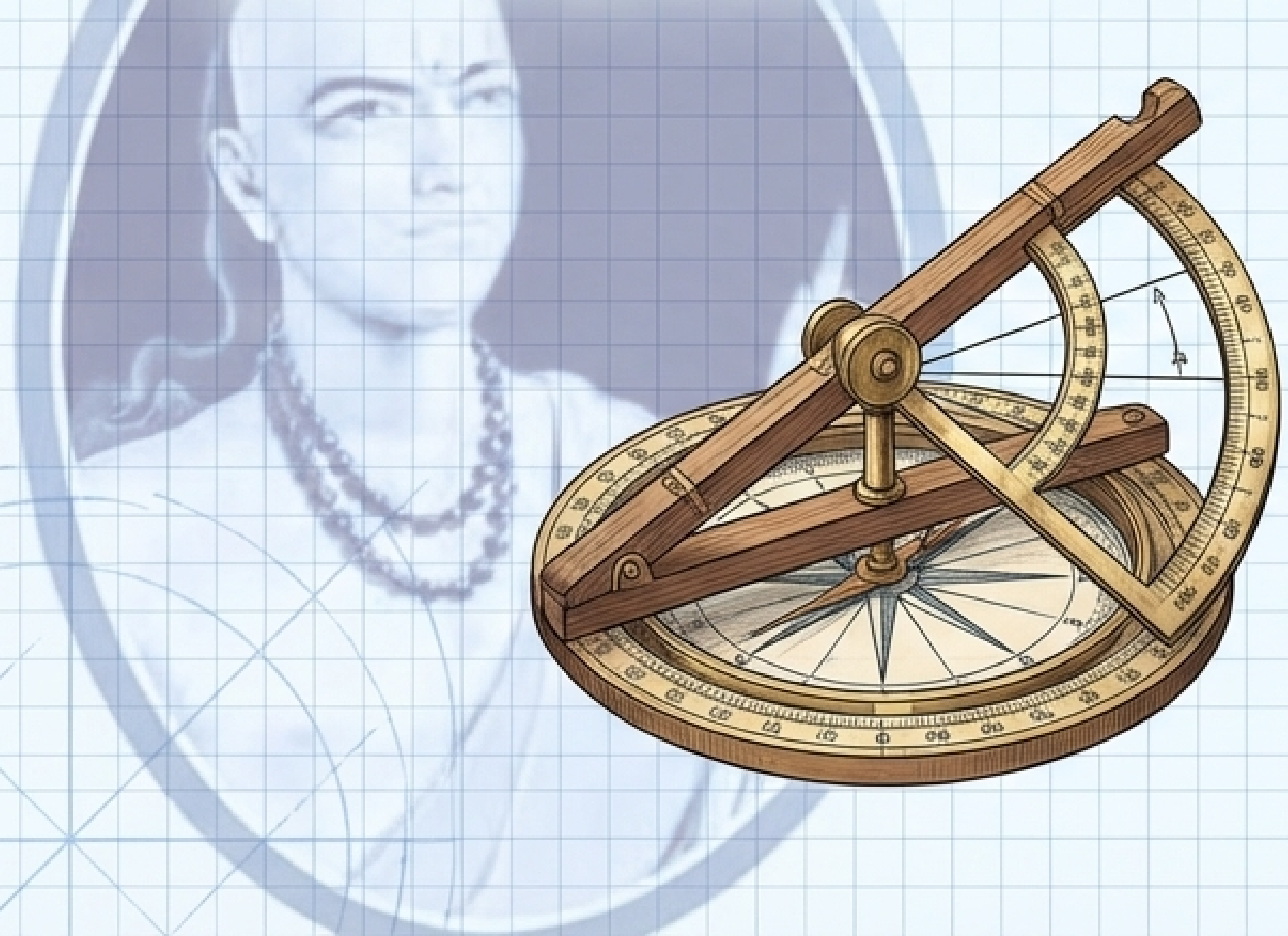
# The Blueprint of Cycles

Transitioning from Static Triangles to Dynamic Waves: A Visual Guide to Circular Trigonometry.



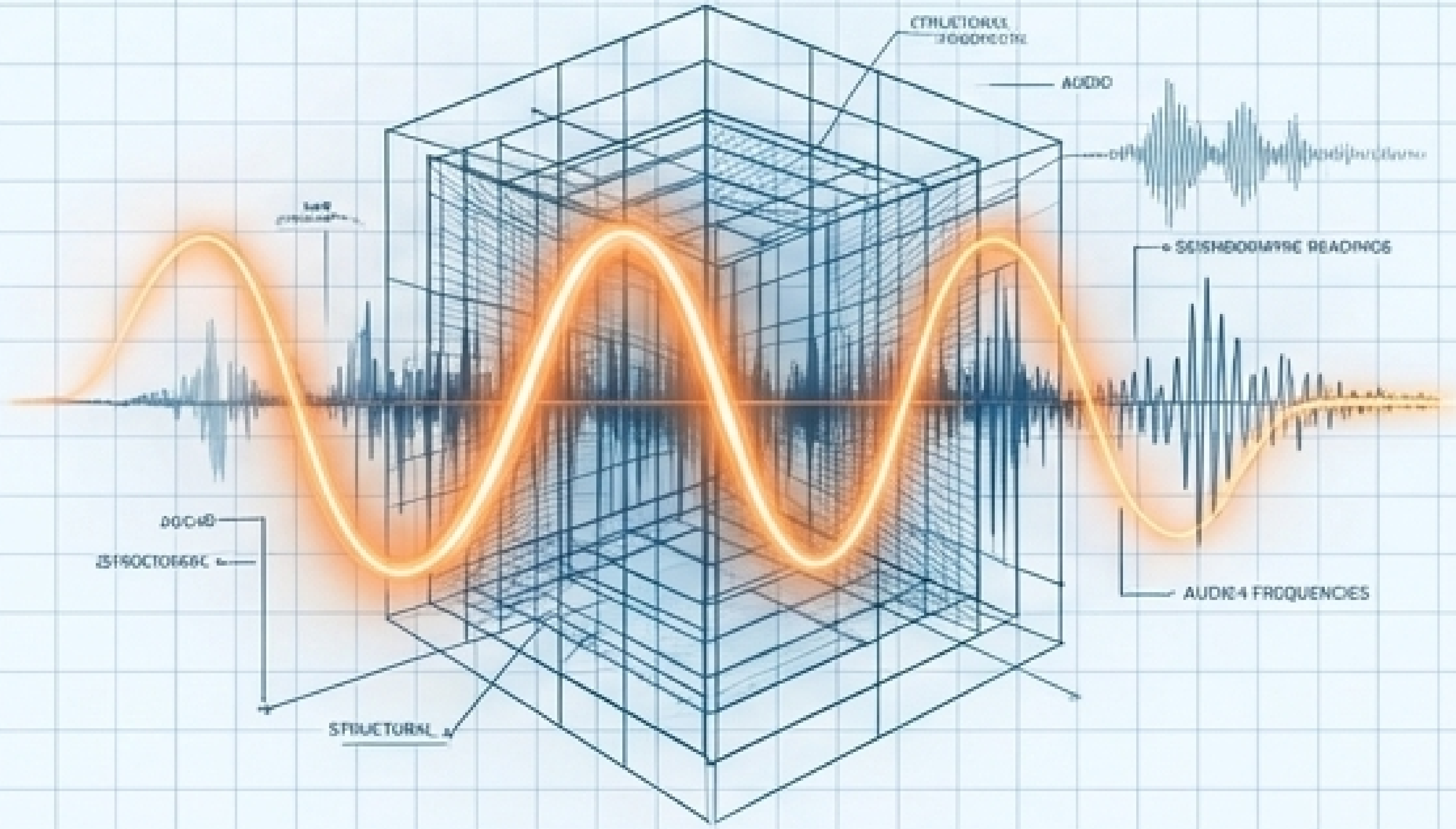
# Trigon + Metron

- Measuring Triangles
- Used by ancient sea captains, surveyors, and pioneers for geometric problem-solving.

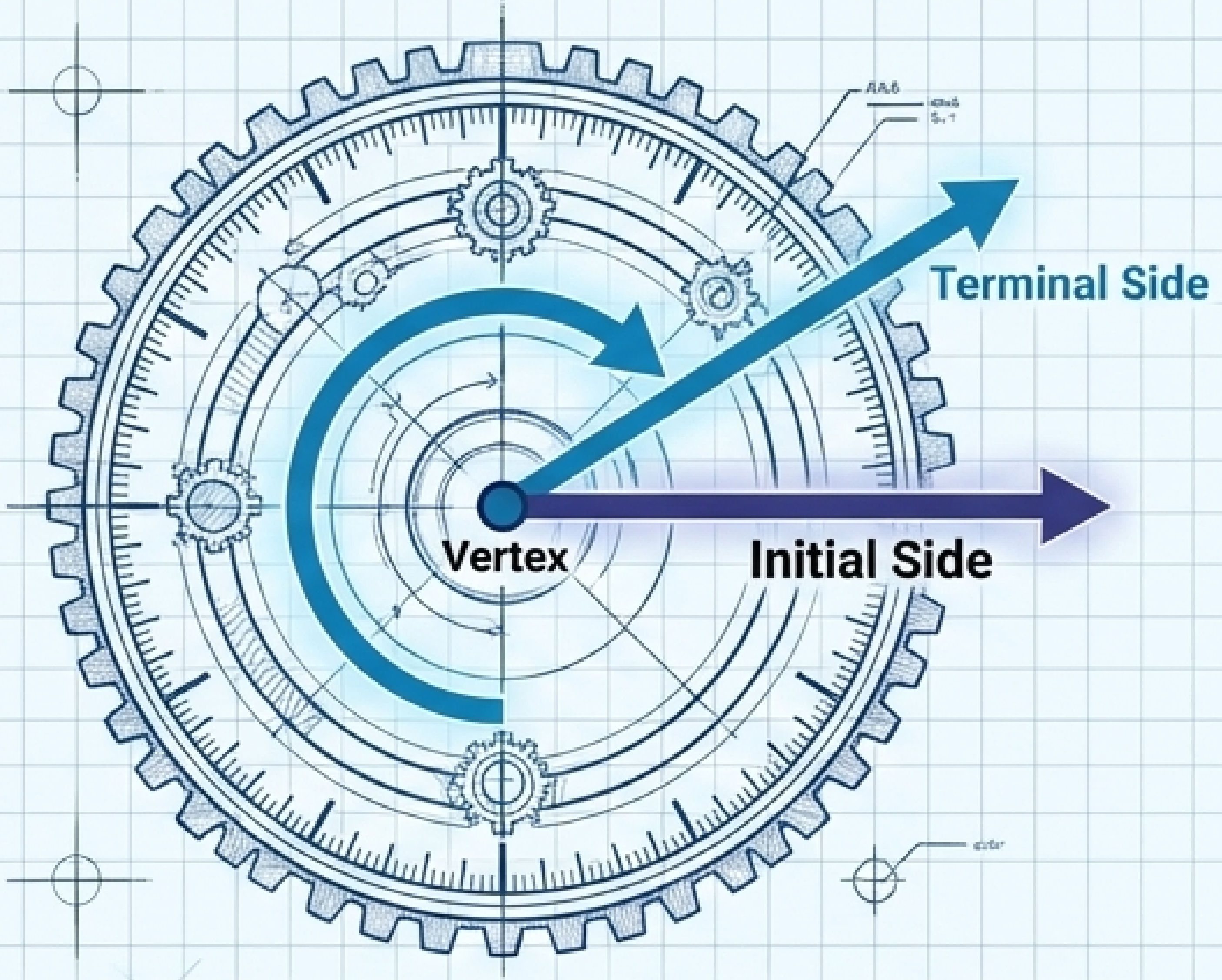


# The Science of Cycles

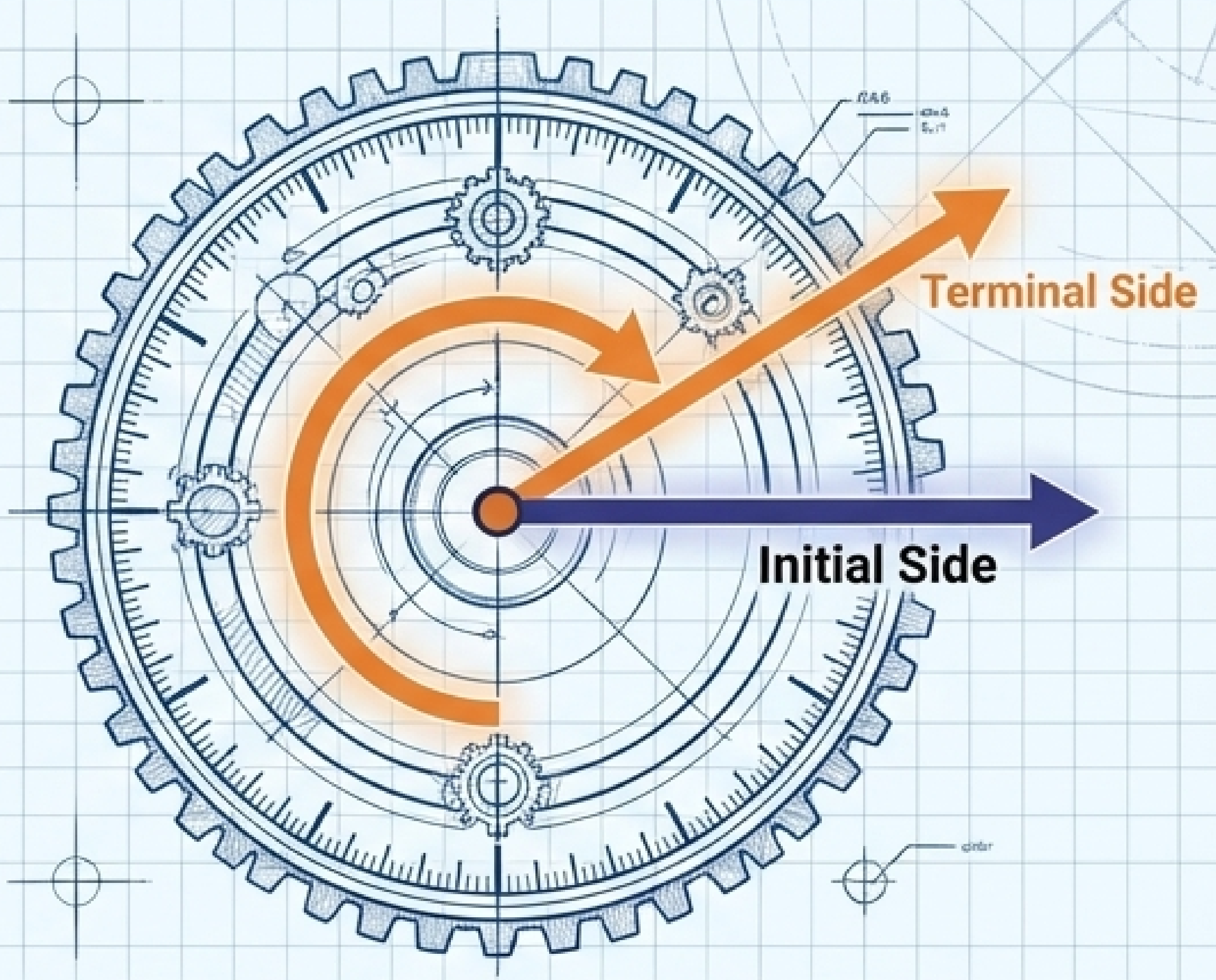
- Modeling Continuous Phenomena
- Predicting ocean tides, designing electric circuits, and analyzing musical tones.



# Redefining the Angle



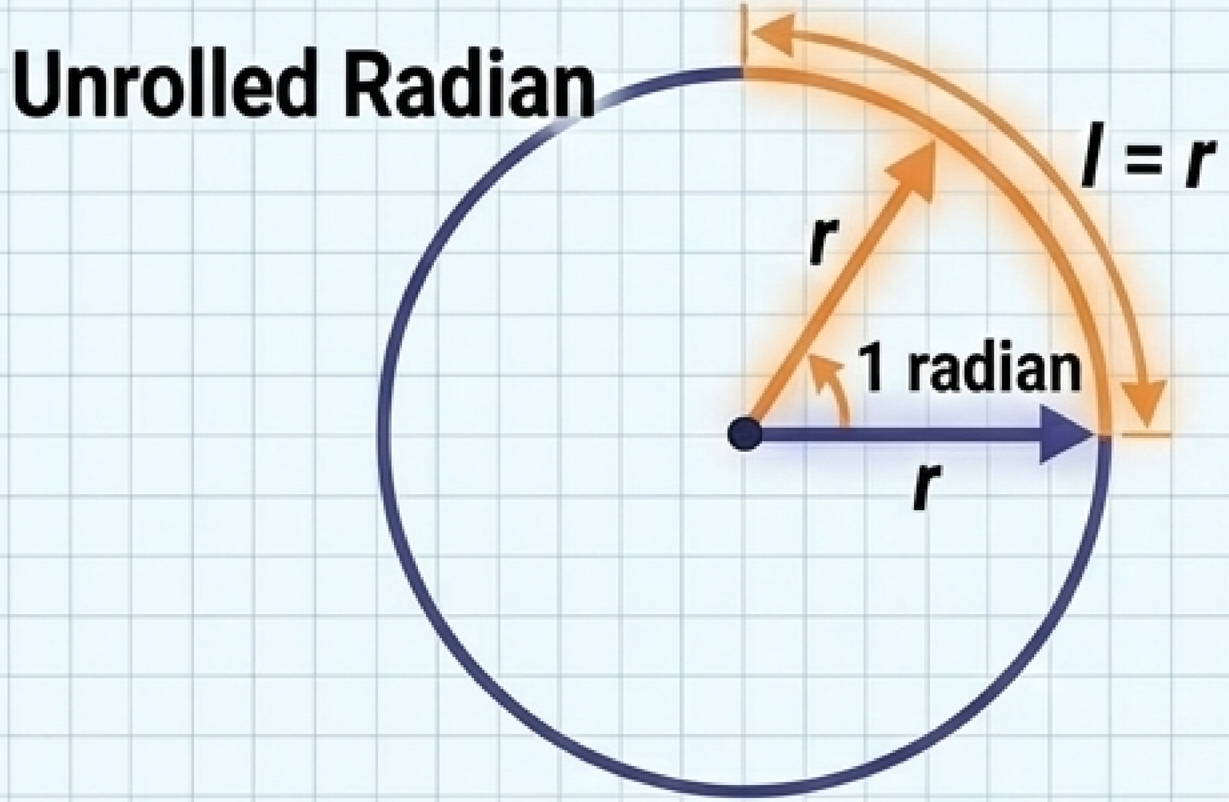

Anticlockwise = Positive Angle

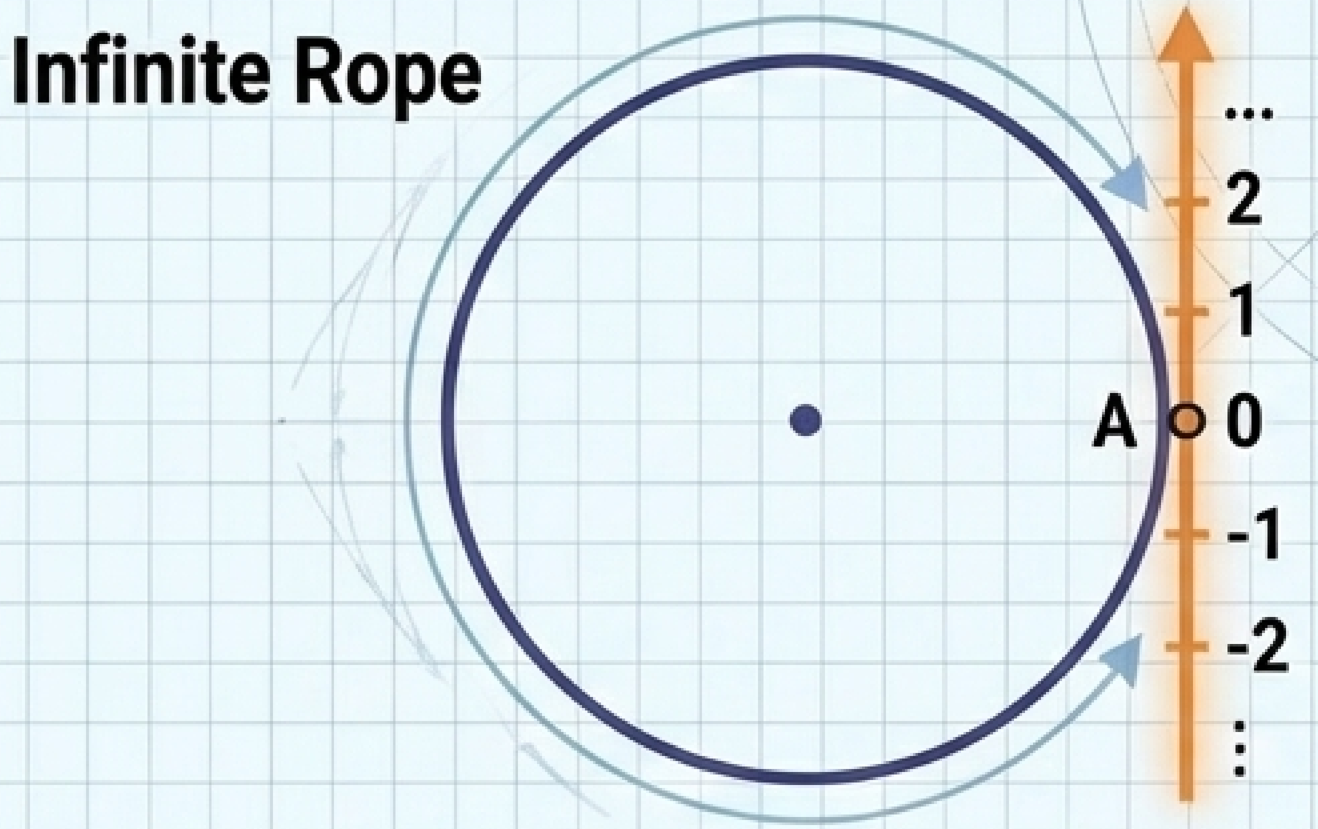



Clockwise = Negative Angle

An angle is no longer just the inside of a polygon.
It is an infinite measure of rotation about a vertex point.

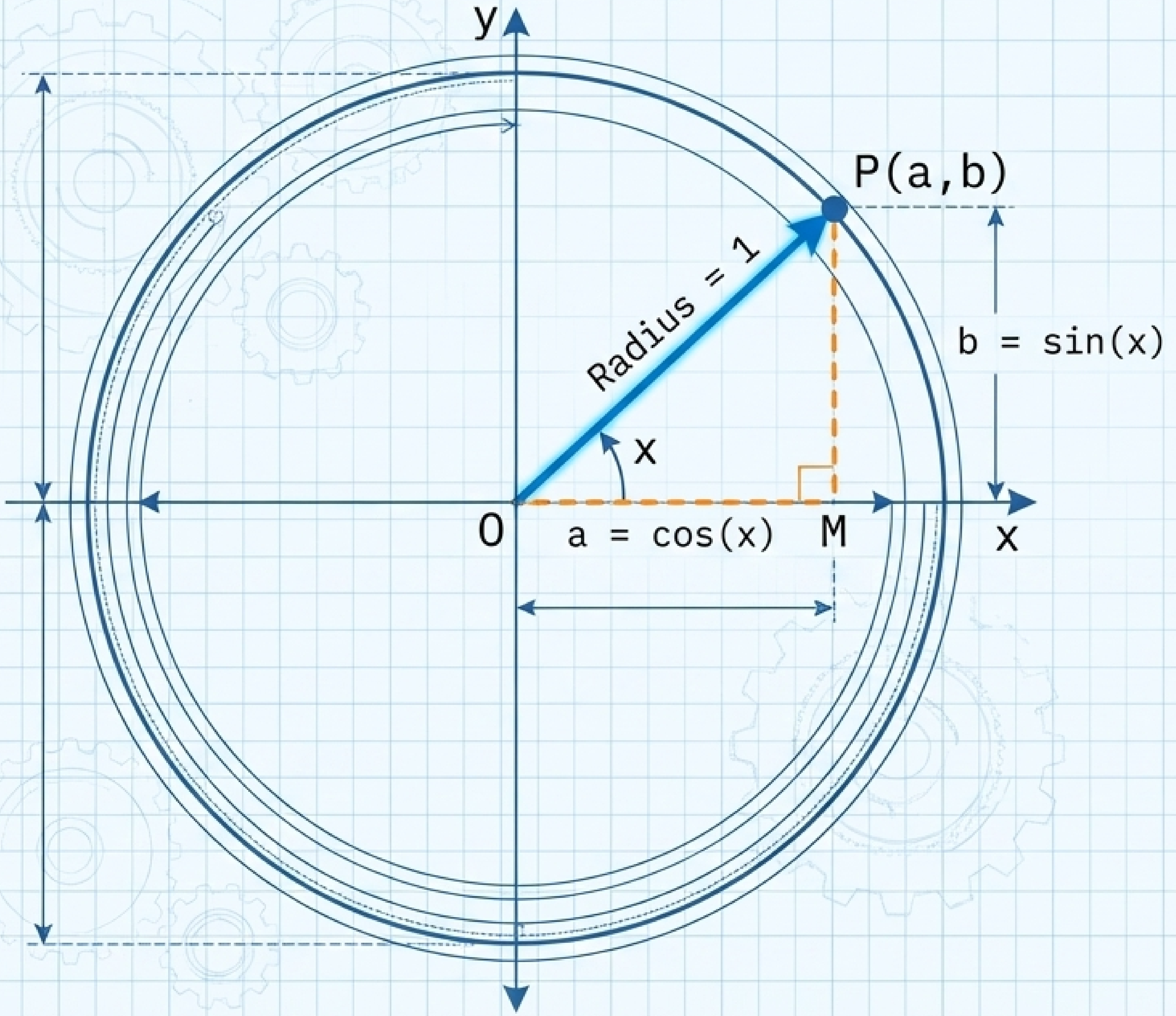
# The Measurement Matrix: Degrees vs. Radians




| Feature | Degrees | Radians |
|---|---|---|
| Base Concept | 1/360th of a revolution | Arc length equal to radius ($l = r$) |
| The Master Ratio | 180° | π radians |
| Conversion | To Degrees: × 180/π | To Radians: × π/180 |
| Use Case | Navigation & surveying | Pure mathematics & mapping continuous real numbers |

# The Unit Circle & Pythagorean Identity



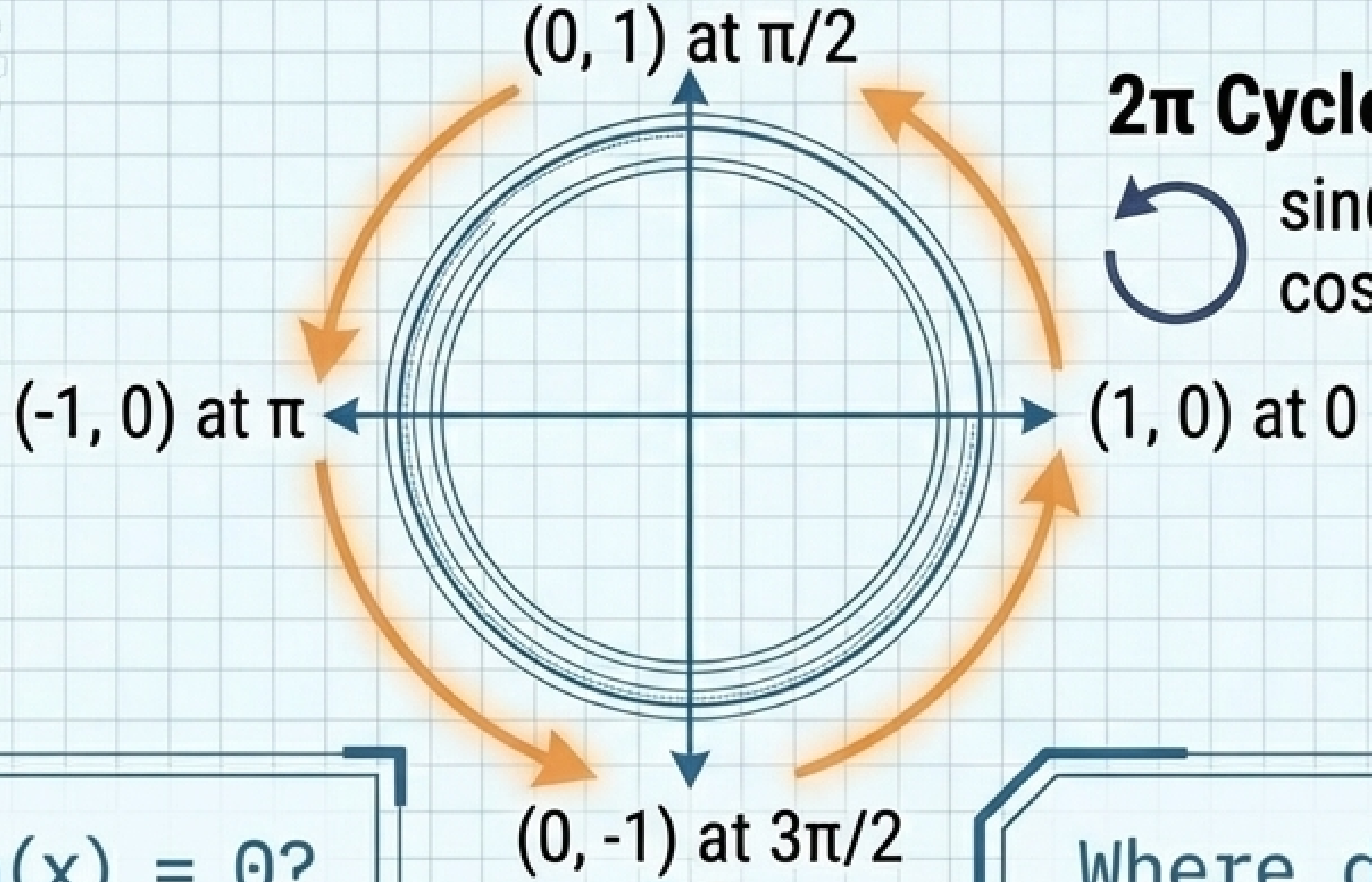

## The Pythagorean Proof

$$OM^2 + MP^2 = OP^2$$

$$a^2 + b^2 = 1$$

$$\cos^2(x) + \sin^2(x) = 1$$

# Zeros, Limits, and Periodicity




**2π Cycle**

$\sin(2n\pi + x) = \sin(x)$
$\cos(2n\pi + x) = \cos(x)$

`Where does sin(x) = 0?`

The y-coordinate is 0 at horizontal poles.

Equation: $\mathbf{x = n\pi}$.

`Where does cos(x) = 0?`

The x-coordinate is 0 at vertical poles.

Equation: $\mathbf{x = (2n + 1)\pi/2}$.

# Signs of Trigonometric Functions

π/2 to π

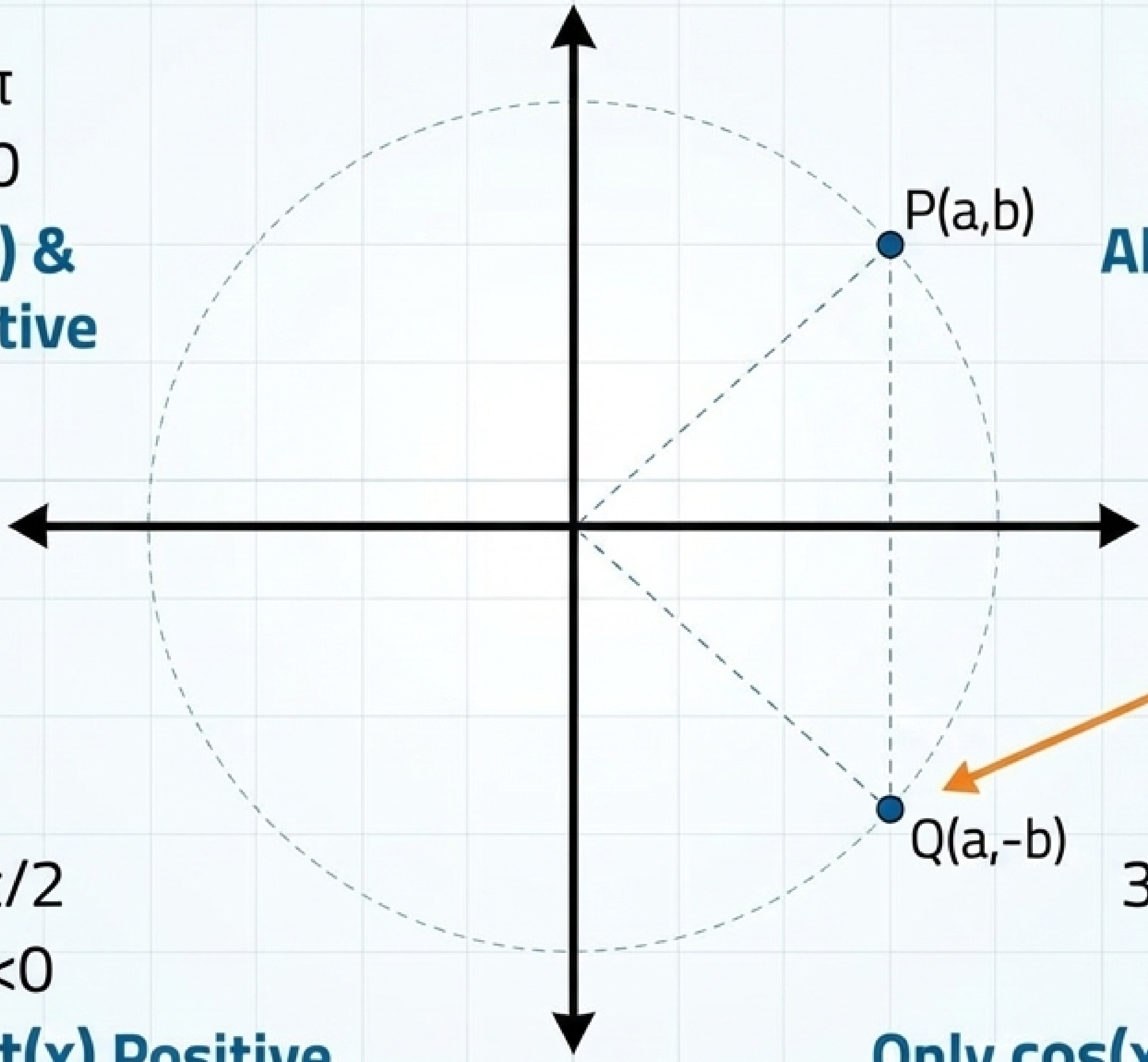
a<0, b>0

**Only sin(x) & csc(x) Positive**

0 to π/2

a>0, b>0

**All functions Positive**




**Negative Angle Rule:** Rotating negatively drops into Quad IV. The x-value stays the same, but y inverts. Therefore: **cos(-x) = cos(x)** and **sin(-x) = -sin(x).**

π to 3π/2

a<0, b<0

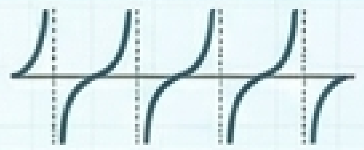
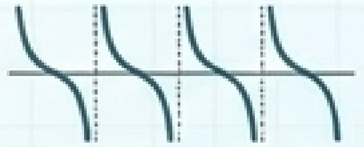
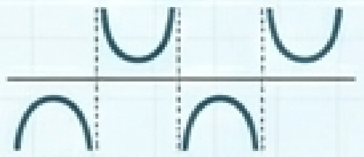
**Only tan(x) & cot(x) Positive**

3π/2 to 2π

a>0, b<0

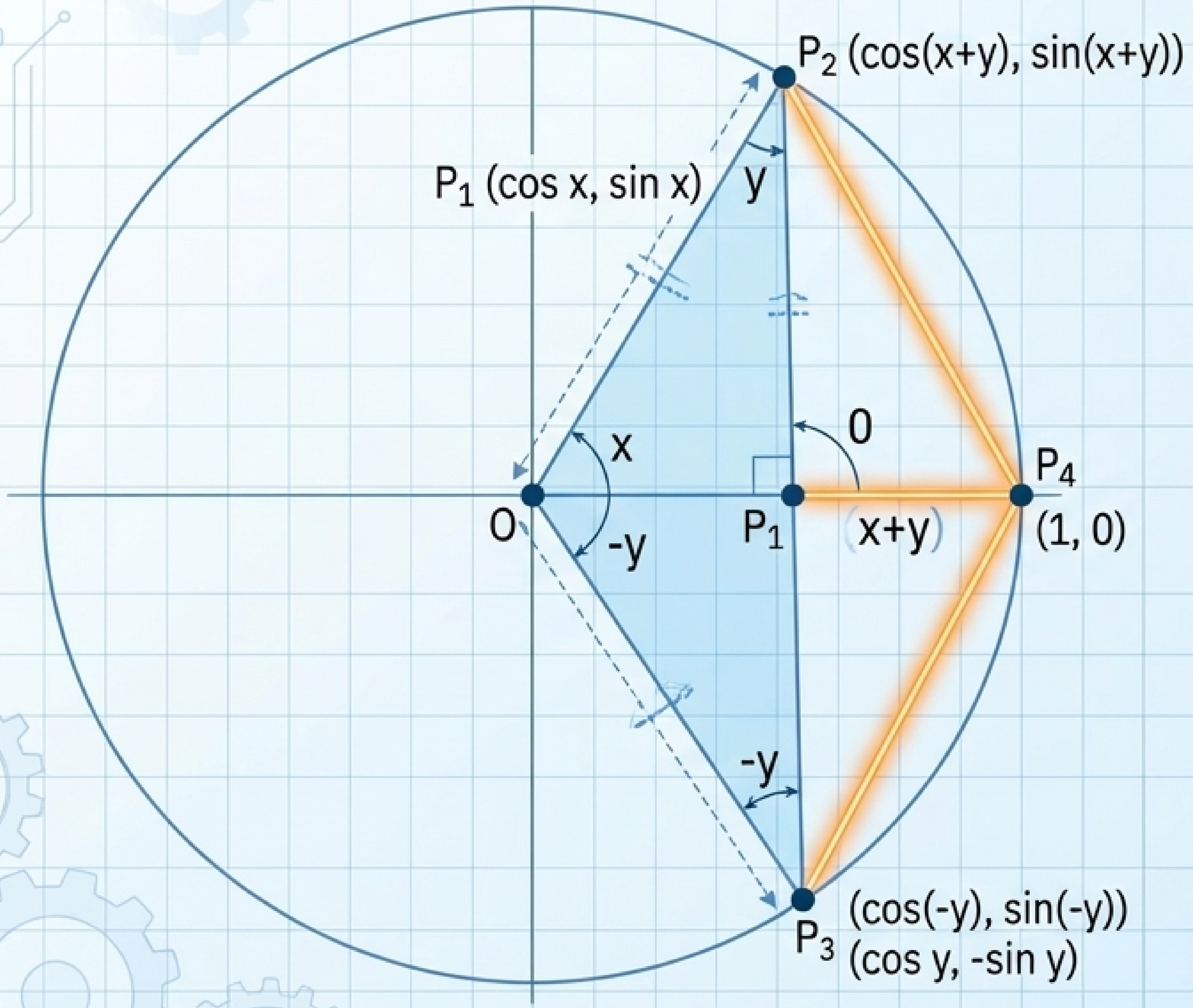
**Only cos(x) & sec(x) Positive**

# Function Limit Matrix: Domain & Range

| Function | Domain (Valid Inputs) | Range (Outputs) | Sparkline |
|---|---|---|---|
| sin(x) | All Reals (R) | [-1, 1] |  |
| cos(x) | All Reals (R) | [-1, 1] |  |
| tan(x) | R except odd multiples of π/2 | All Reals (R) |  |
| cot(x) | R except multiples of π | All Reals (R) |  |
| sec(x) | R except odd multiples of π/2 | y ≤ -1 or y ≥ 1 |  |
| csc(x) | R except multiples of π | y ≤ -1 or y ≥ 1 |  |

Note: Infinity isn't a number—it's a behavior. When tan(x) approaches π/2, it assumes arbitrarily large positive values, denoted as ∞.

# Geometric Proof of the Master Identity




### Step 1: Congruence & Chord Equality

Because the interior triangles (P1OP3 & P2OP4) are perfectly congruent due to identical angle rotation, chord length P1P3 = P2P4.

### Step 2: Distance Formula for P1P3

Using the distance formula:

$$P1P3^2 = [\cos x - \cos(-y)]^2 + [\sin x - \sin(-y)]^2$$

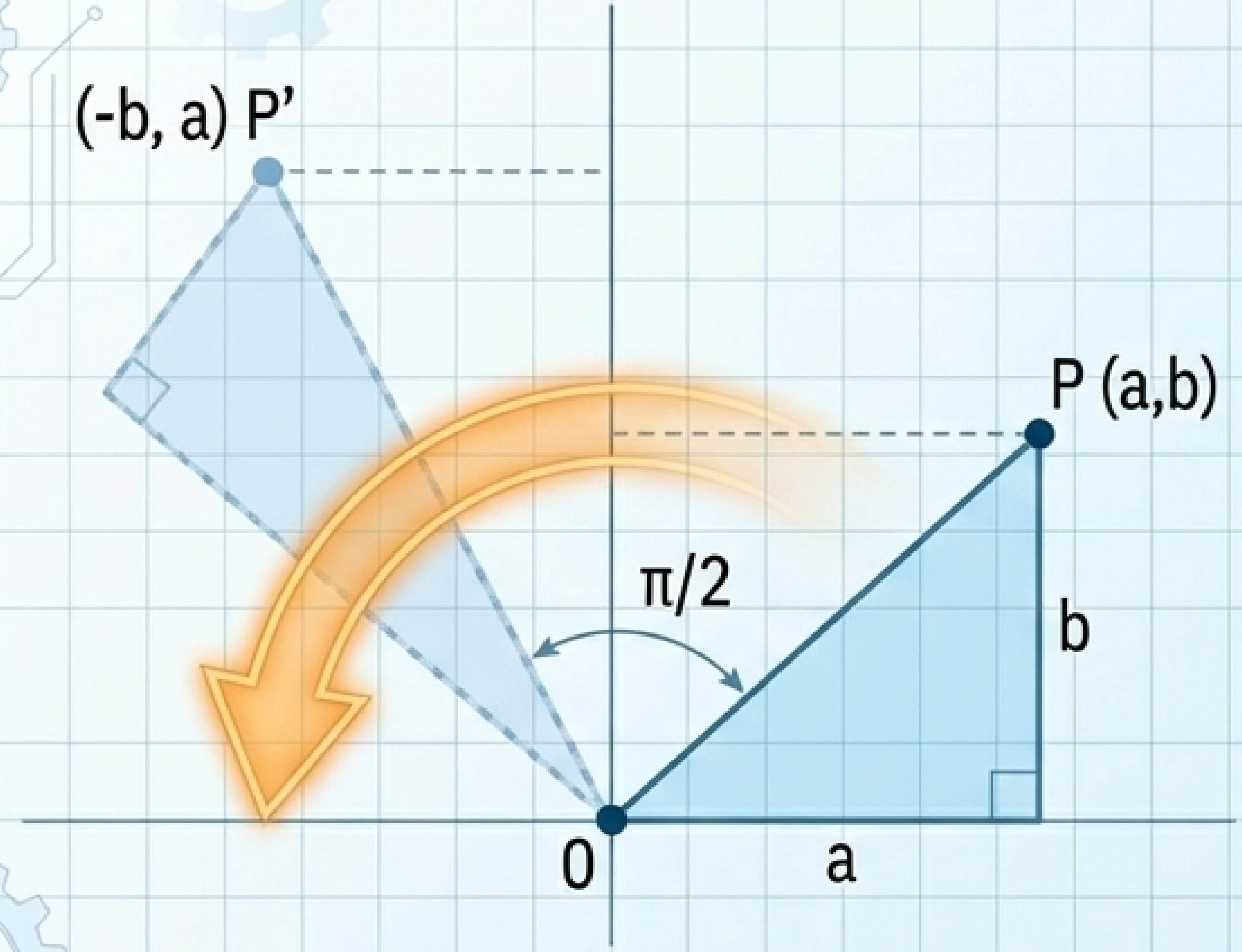
### Step 3: Distance Formula for P2P4

$$P2P4^2 = [\cos(x+y) - 1]^2 + [\sin(x+y) - 0]^2$$

Equating the two chord lengths (and their squares) yields the master identity:

$$\cos(x+y) = \cos x \cos y - \sin x \sin y$$

# Geometric Proof: Rotation & Reflection Identities




**Synthesis Matrix**

| Rule | Why? |
|---|---|
| $\cos(\pi/2 - x) = \sin(x)$ & $\sin(\pi/2 - x) = \cos(x)$ | A 90° rotation physically flips the triangle; horizontal width becomes vertical height. |
| $\cos(\pi - x) = -\cos(x)$ & $\sin(\pi - x) = \sin(x)$ | A 180° flip perfectly mirrors the triangle across the y-axis. |

Every trig**onometric identity** is simply a **geographic fact** about the unit circle. **Master** the **geometry**, and the **algebra** solves itself.