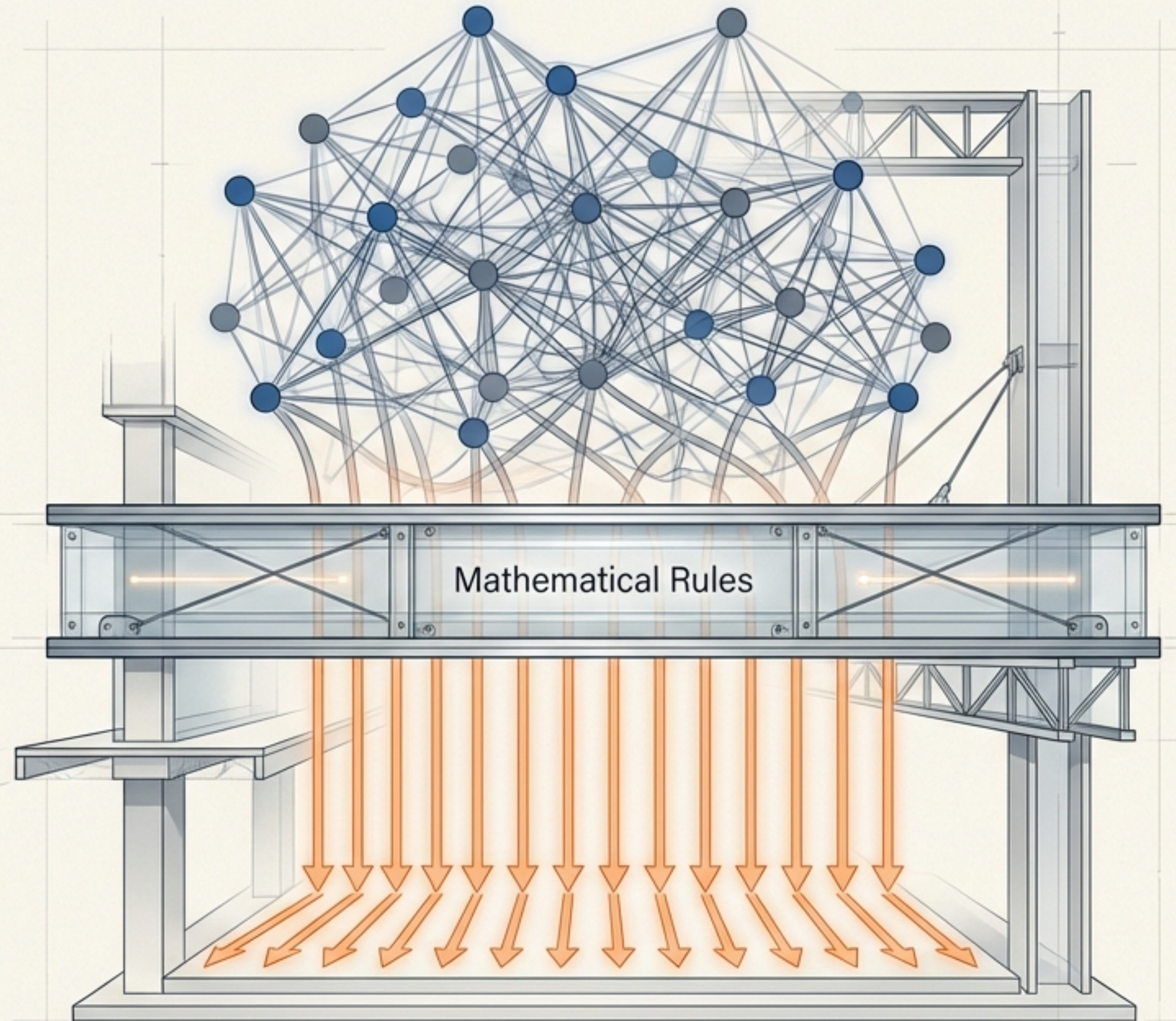


The Architecture of Predictability

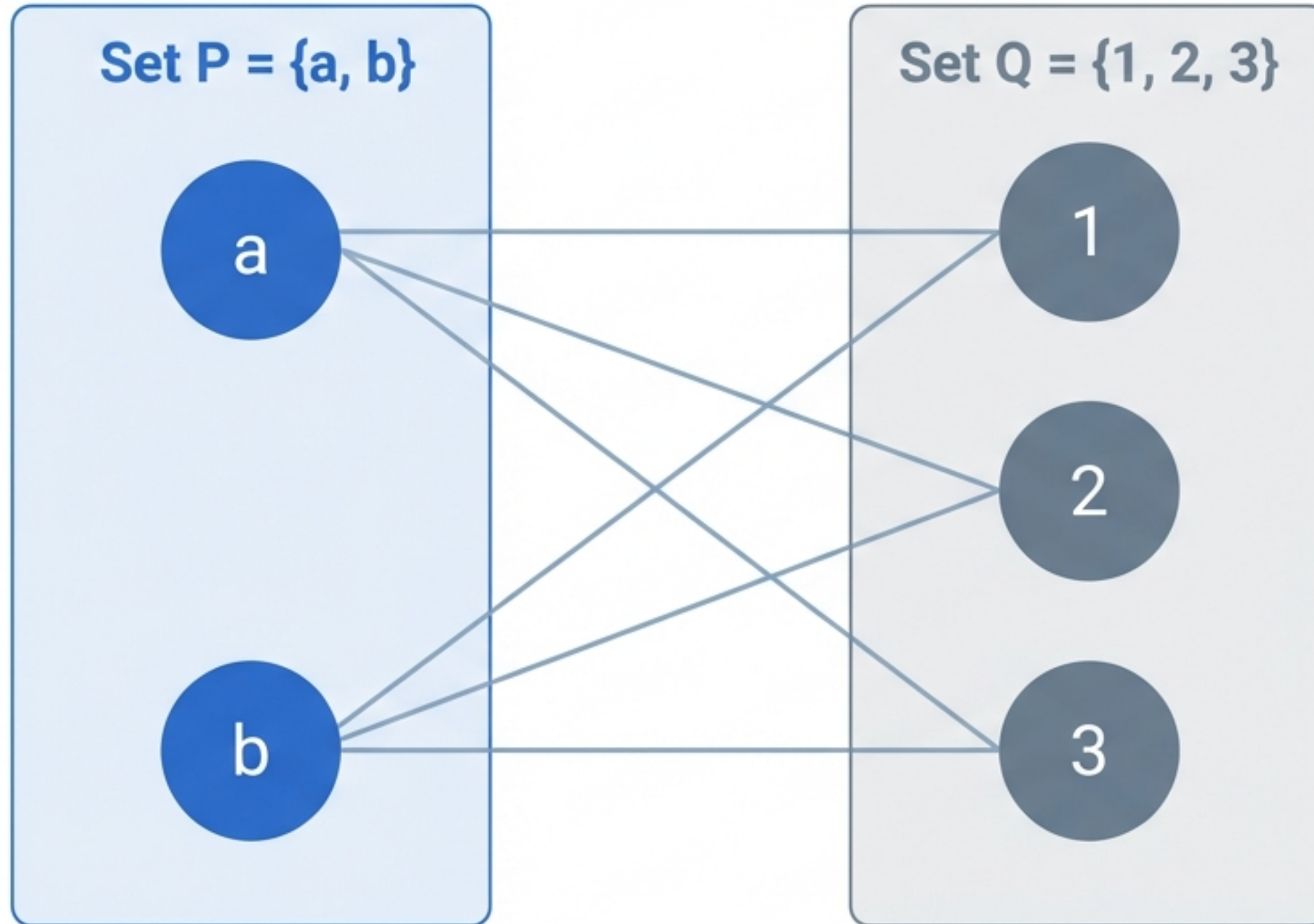
Understanding Sets, Relations,
and the Algebra of Functions

Mathematics is the indispensable
instrument of all physical research.
– *Berthelot*



The Cartesian Product: Zero Constraints

Dense Node Network



Key Concept Block

Definition

$P \times Q$ is the set of all possible ordered pairs (p, q) where $p \in P$ and $q \in Q$.

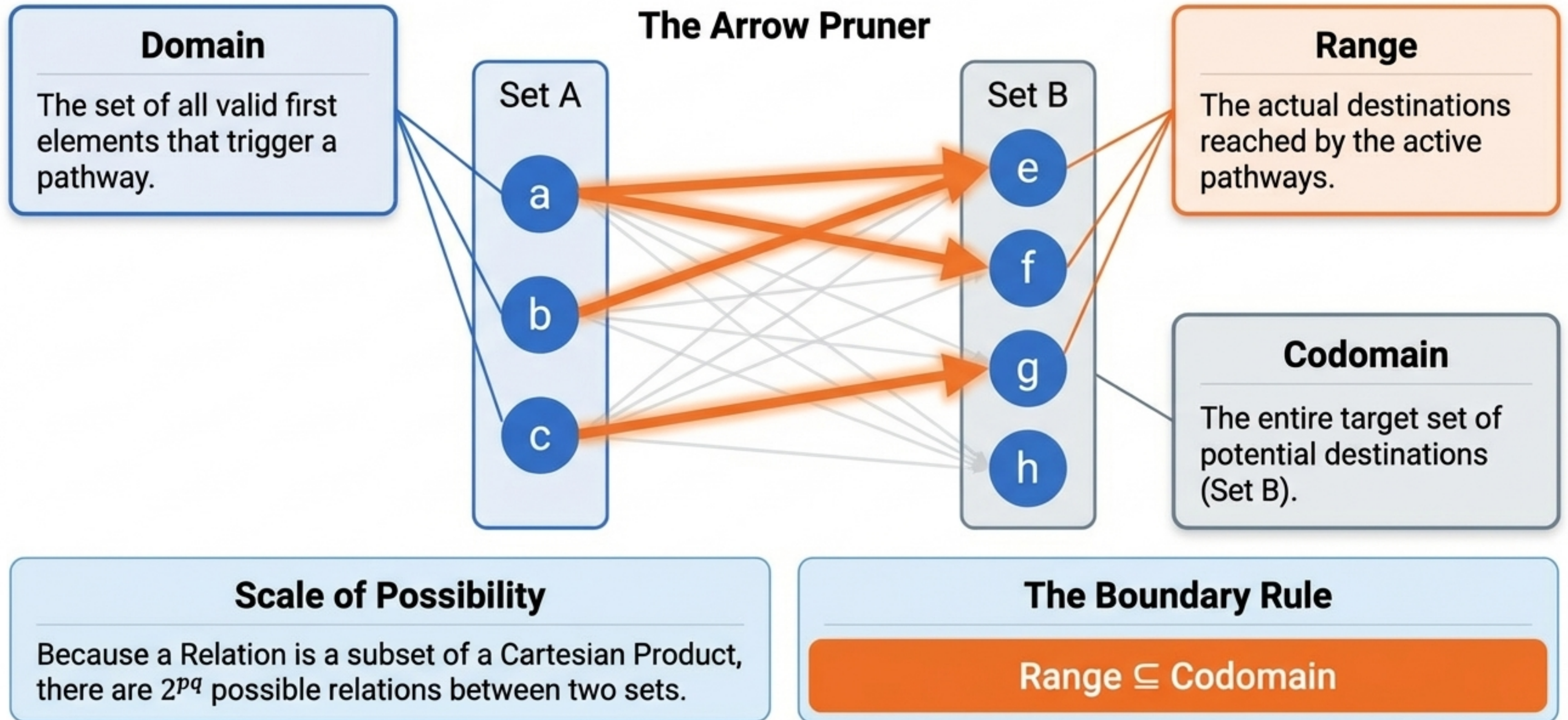
The Rule of Scale

If $n(P) = p$ and $n(Q) = q$, then the total number of pairs $n(P \times Q) = pq$.



Equality of pairs is **absolute**:
 (a, b) is never equal to (b, a) .

Relations: Pruning the Network



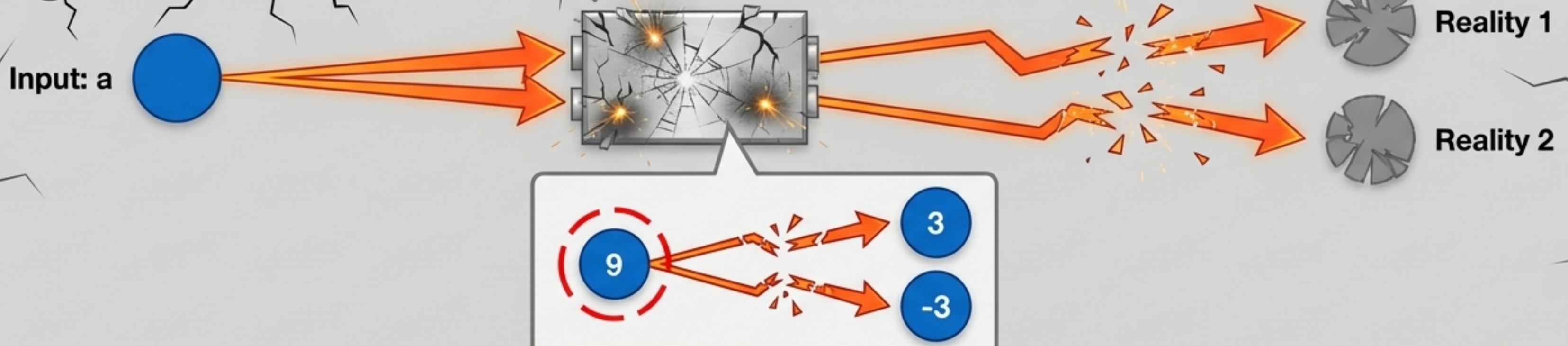
Functions: The Ultimate Mapping Machine

Every element of Set A has **ONE** and **ONLY ONE** image in Set B.

Valid Function

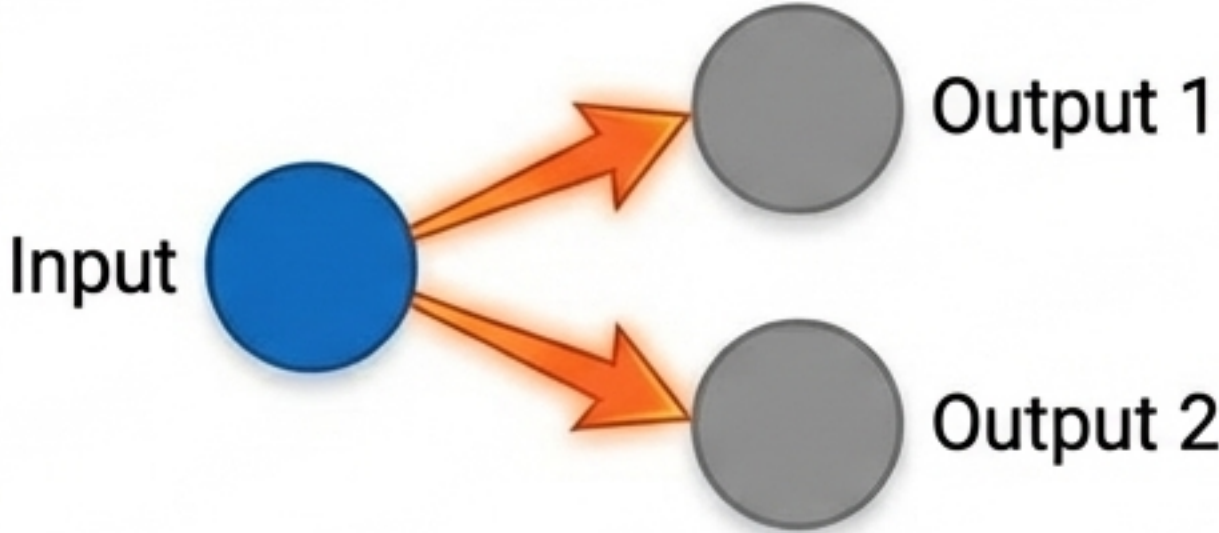
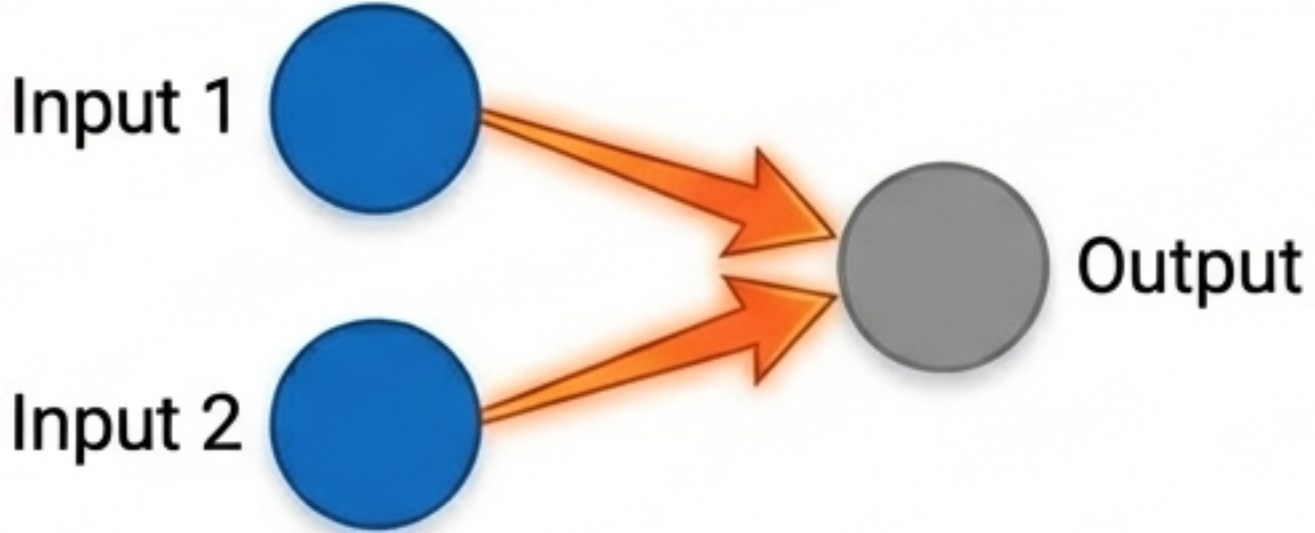


Engine Failure



⚠ System Error: A single input cannot yield multiple realities.

Diagnostic Matrix: Relation vs. Function

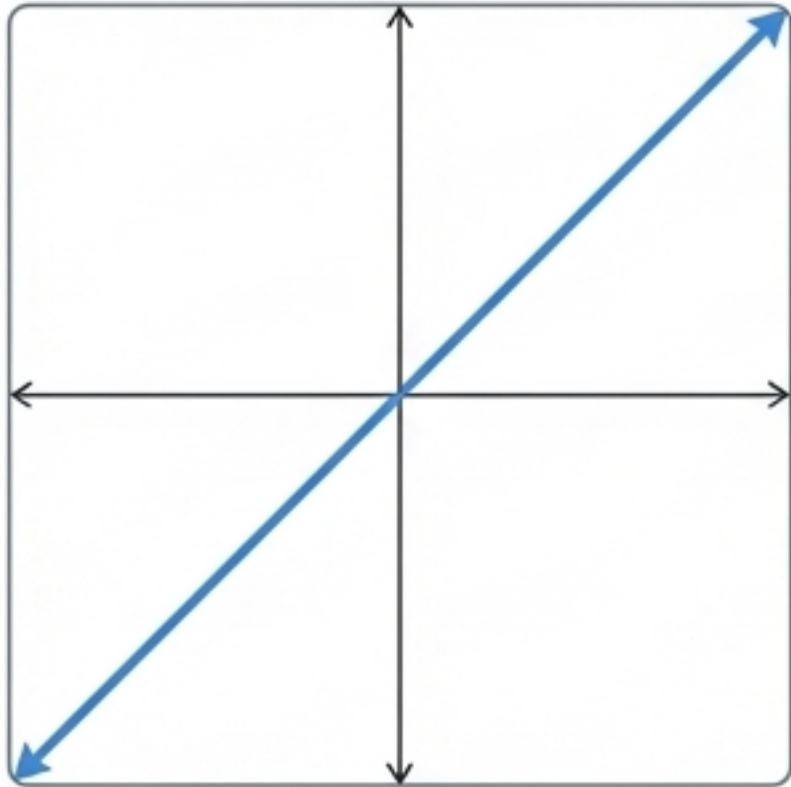
	Relation (Loose Links)	Function (Predictable Mappings)
Visual Architecture		
Constraint Level	Low. One input can have multiple unpredictable outputs.	Absolute. Every input maps to exactly one unique output.
Mathematical Identity	A subset of the Cartesian Product ($A \times B$).	A specialized, highly-restricted Relation.

Diagnostic Note: If $\text{Range} \subseteq \mathbb{R}$, it is a Real-Valued Function. If $\text{Domain} \subseteq \mathbb{R}$ as well, it is a Real Function.

The Pattern Catalog: Baseline Real Functions

Identity

$$f(x) = x$$

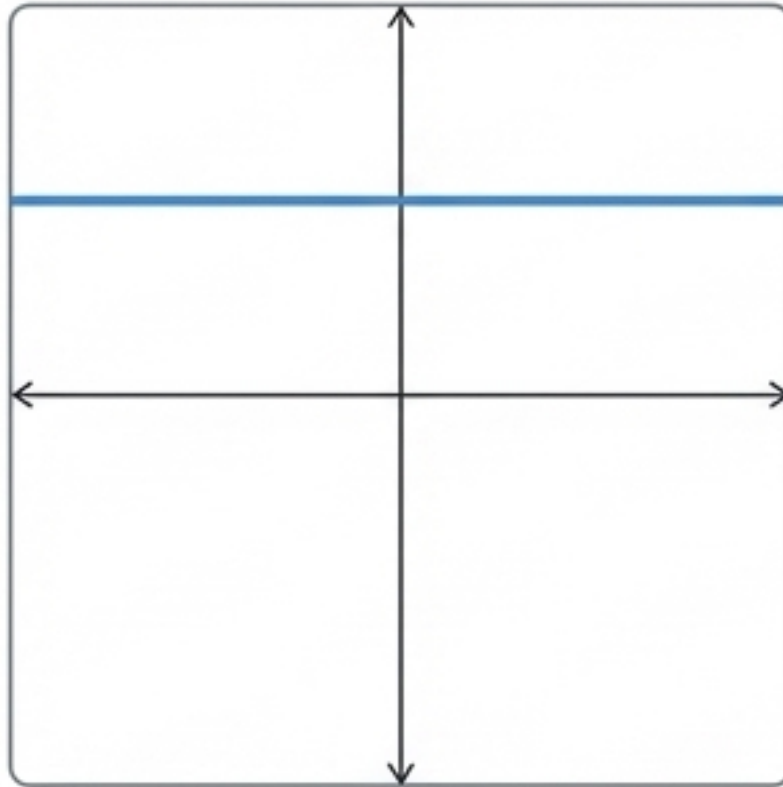


Domain = $\mathbb{R}$

Range = $\mathbb{R}$

Constant

$$f(x) = c$$

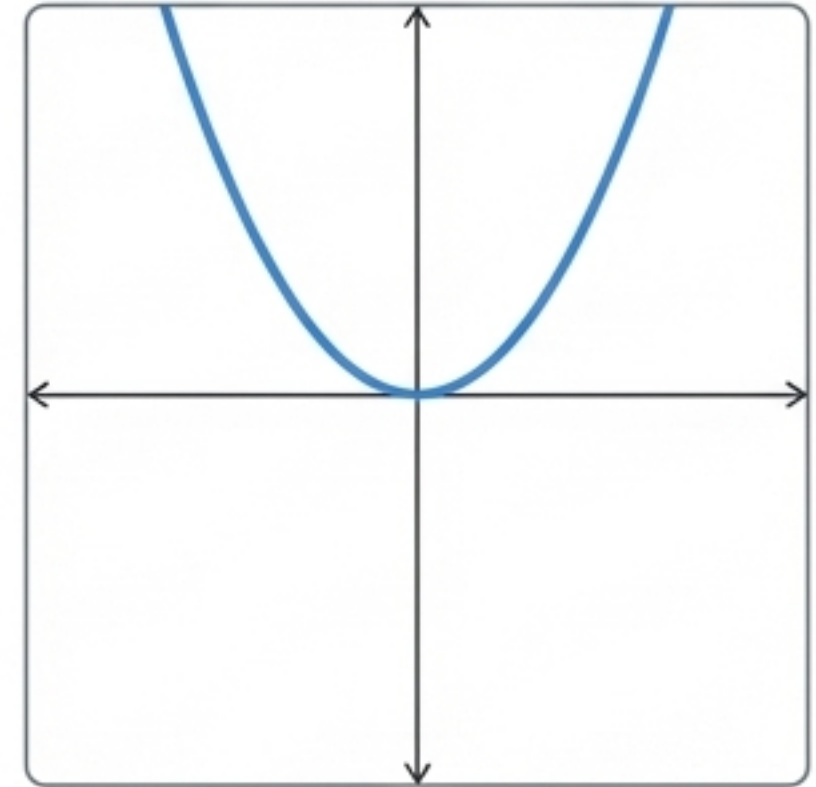


Domain = $\mathbb{R}$

Range = $\{c\}$

Polynomial (Quadratic)

$$f(x) = x^2$$



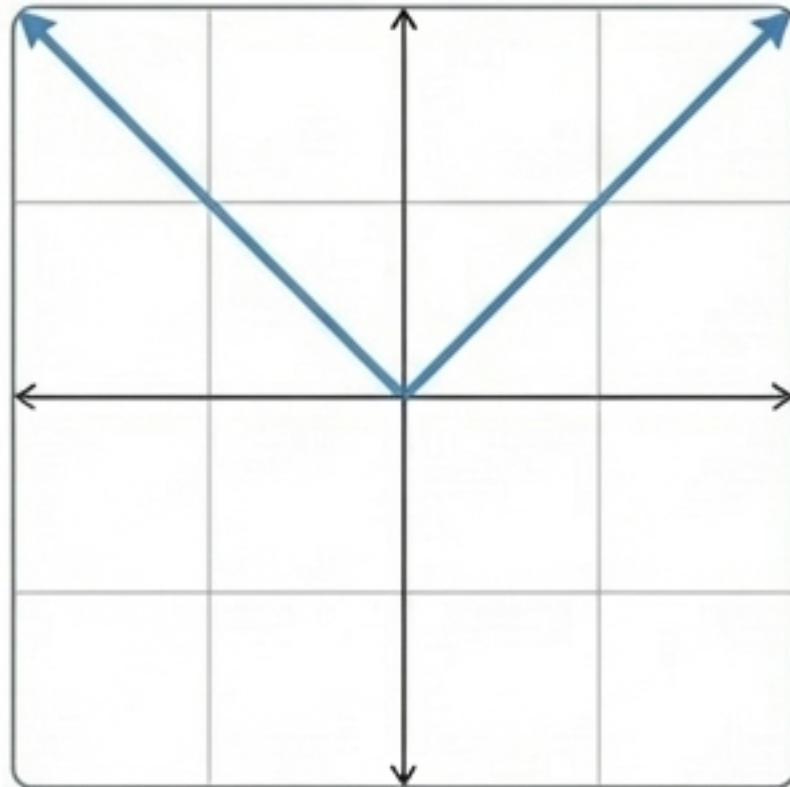
Domain = $\mathbb{R}$

Range = $\{x^2 : x \in \mathbb{R}\}$

The Pattern Catalog: Filters and Snaps

Modulus (Absolute Filter)

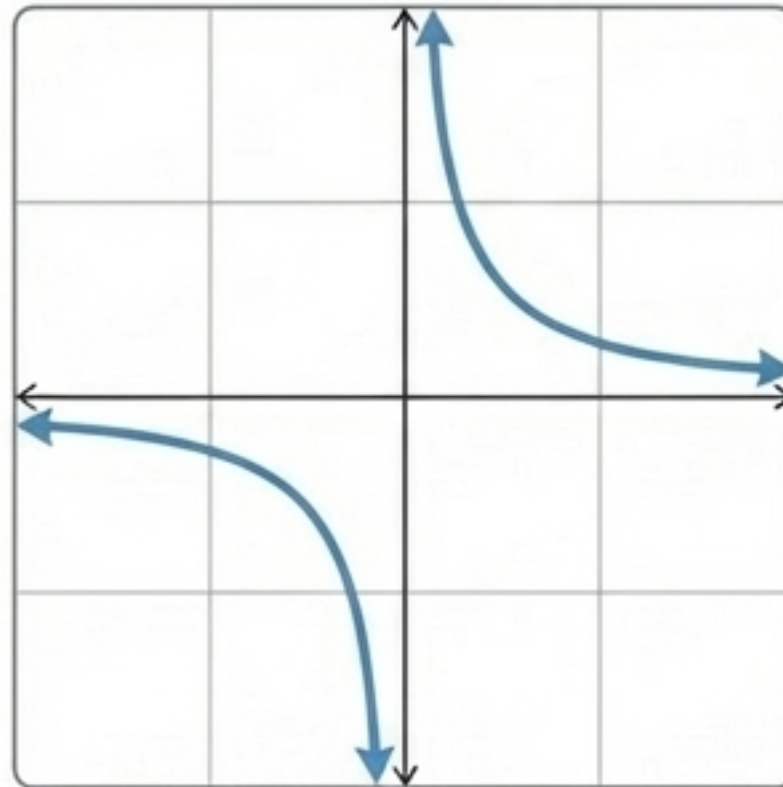
$$f(x) = |x|$$



$$f(x) = x \text{ for } x \geq 0$$
$$f(x) = -x \text{ for } x < 0$$

Rational (Asymptotic Divider)

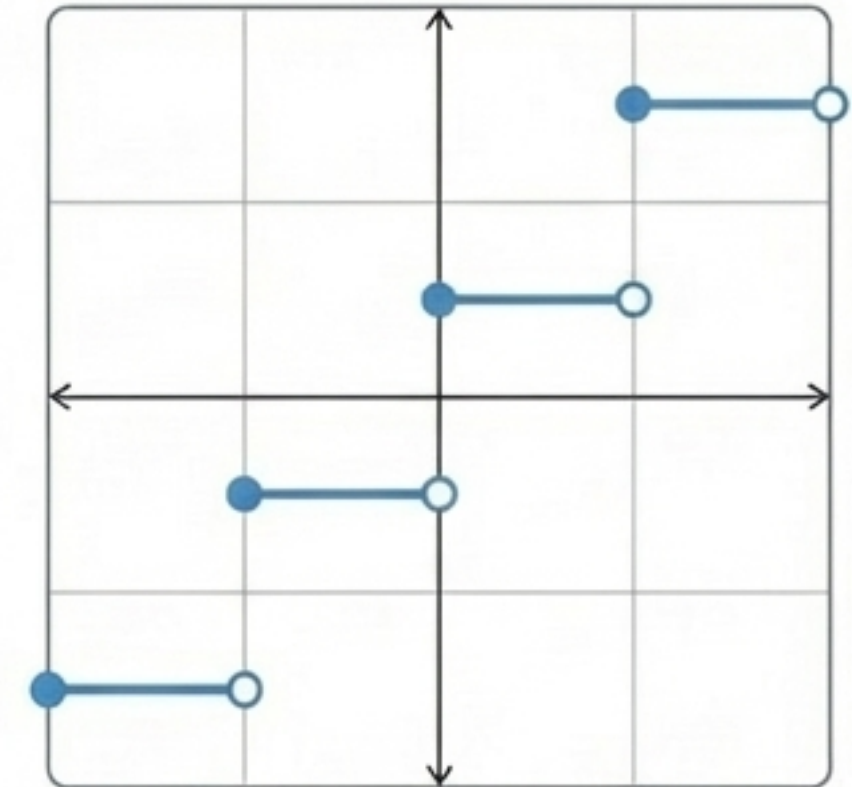
$$f(x) = \frac{1}{x}$$



$$\text{Domain} = \mathbb{R} - \{0\}$$

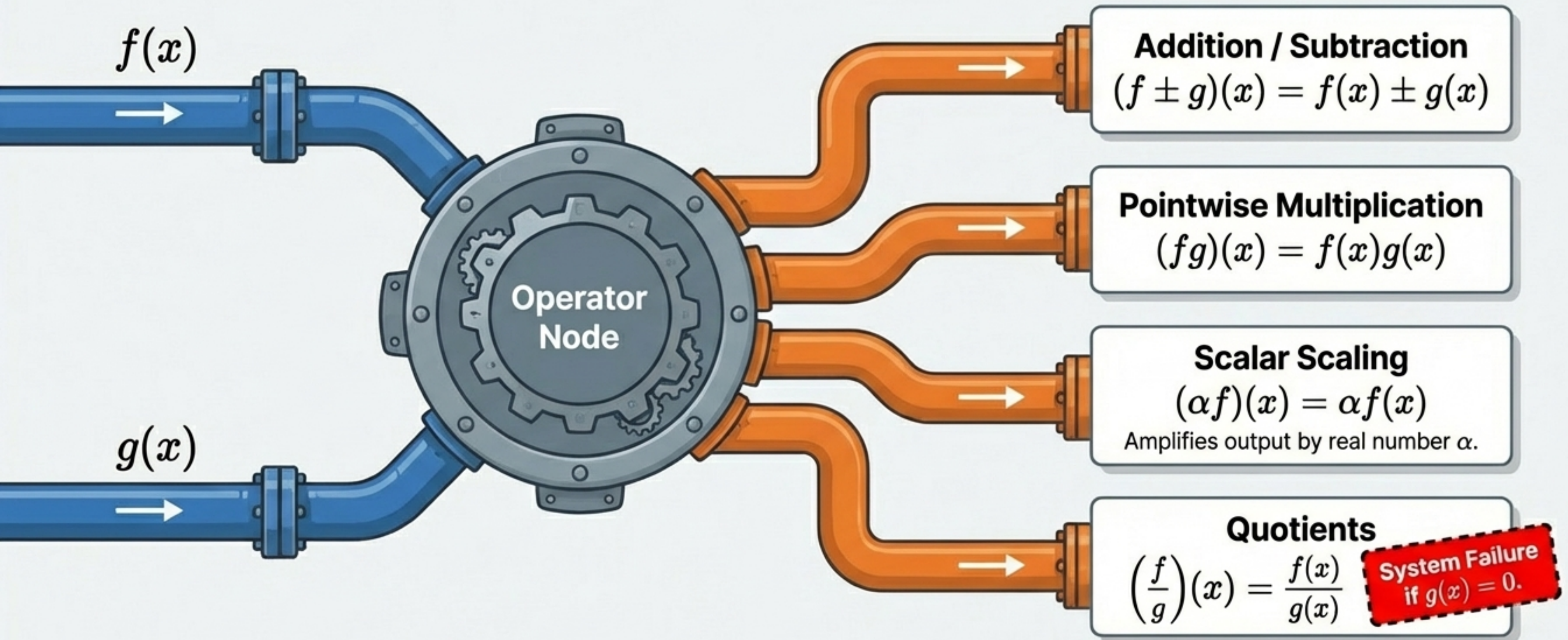
Greatest Integer (Staircase Snap)

$$f(x) = \lfloor x \rfloor$$

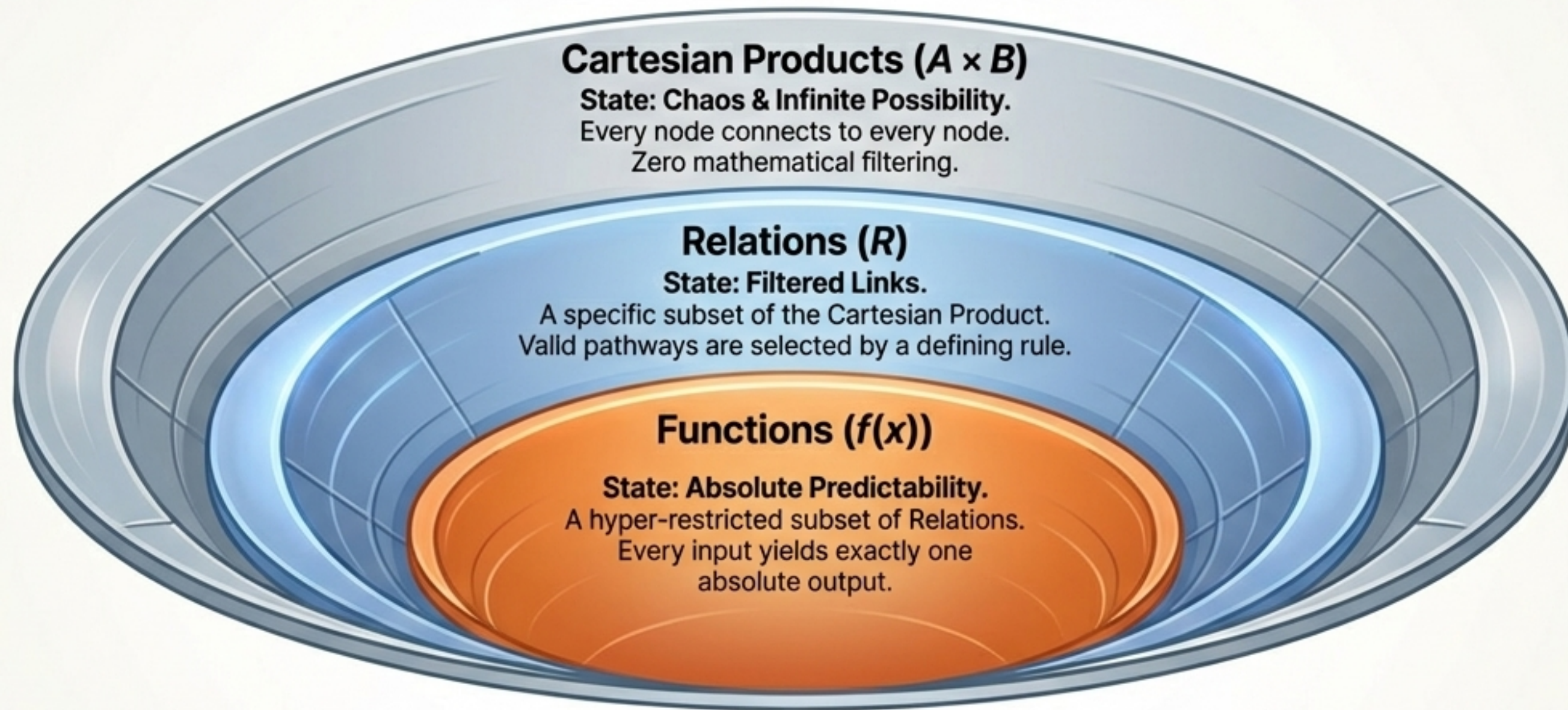


Real numbers “snap” down to the nearest integer floor.

System Integration: The Algebra of Functions



Synthesis: The Constraint Continuum



**Mathematics builds complex systems not by adding chaos,
but by layering predictable constraints.**
