The Geometry of Similarity

Measuring the unmeasurable, from mountain peaks to moving shadows.

The Unreachable Heights

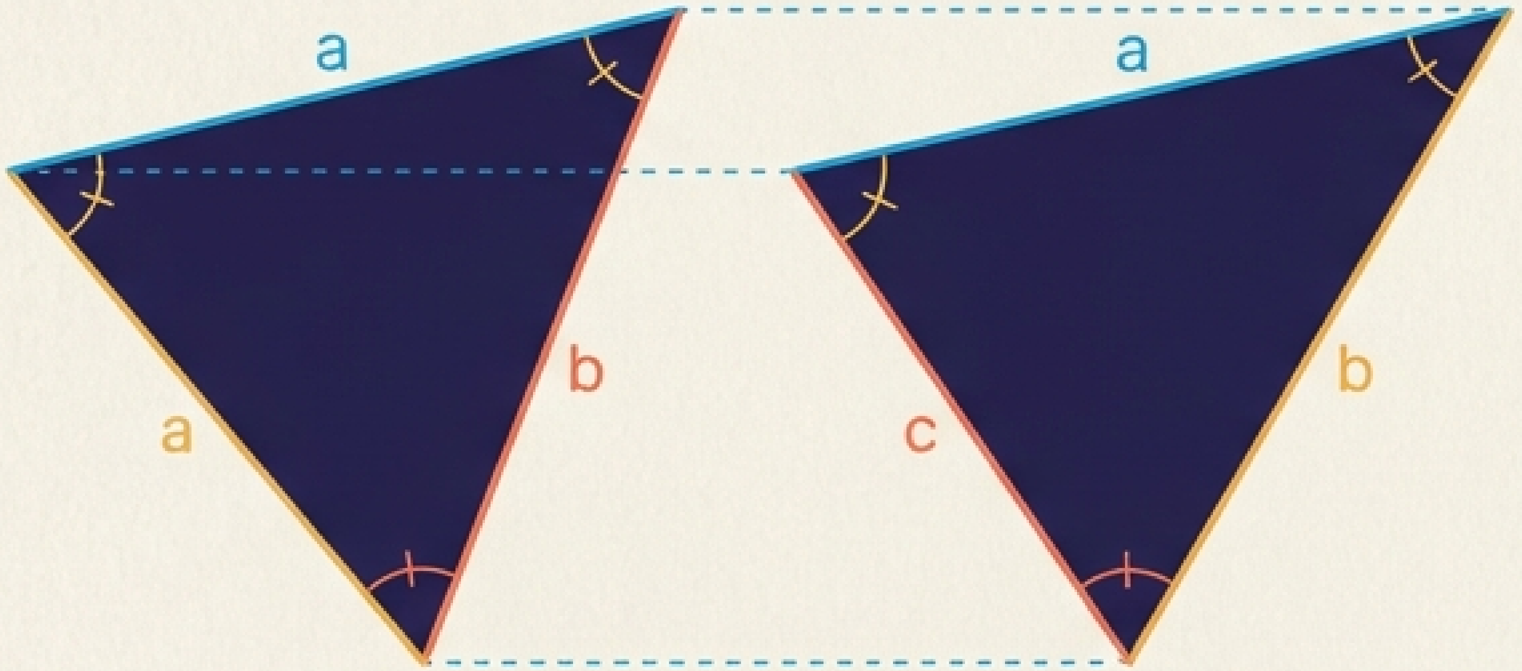
How do we measure the height of Mount Everest or the distance to the Moon without a physical tape measure?

Early mathematicians, like Thales of Miletus, solved this by observing the physical world and recognizing patterns. They discovered that you don't need to reach the top of a mountain to measure it. Instead, you can rely on the principle of indirect measurement—using the known dimensions of accessible objects to calculate the unknown dimensions of unreachable ones.

Congruent vs. Similar

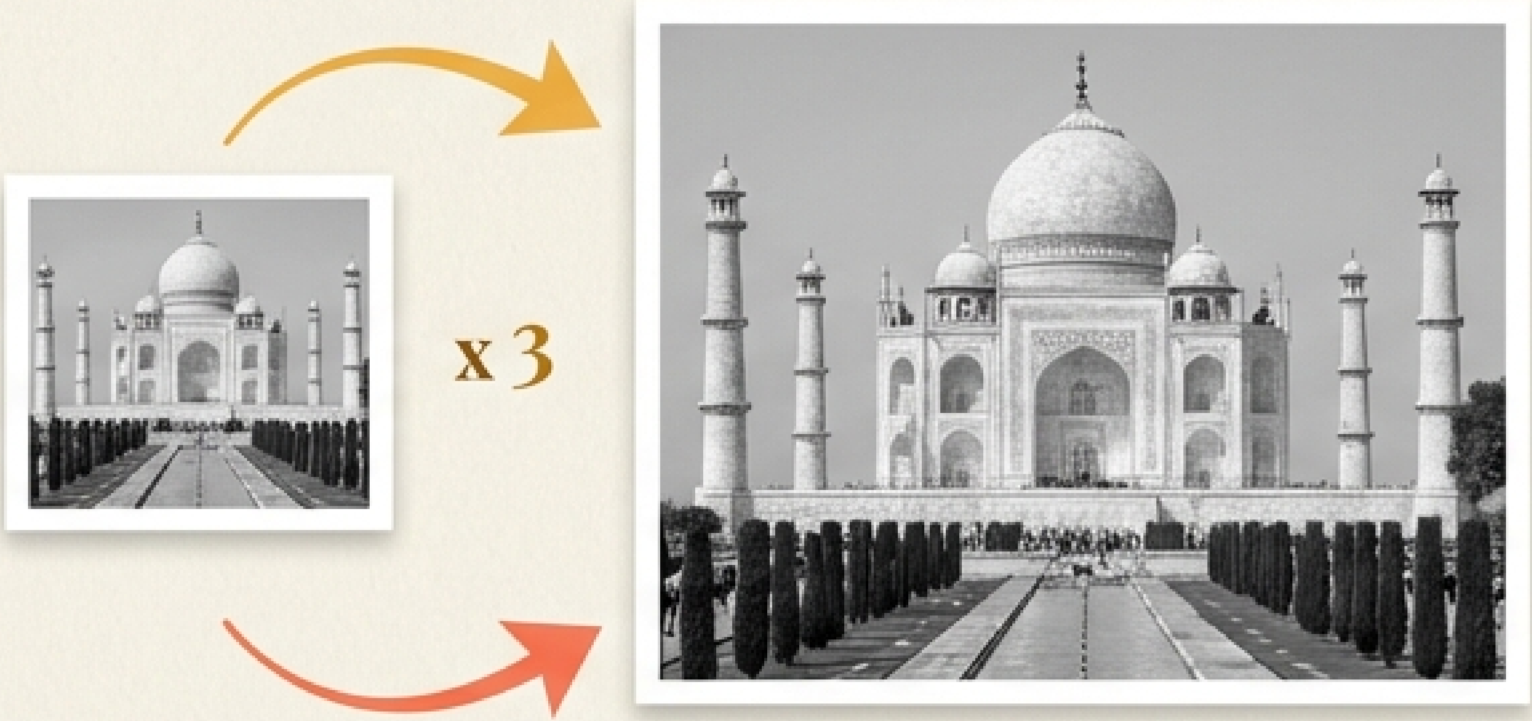
Congruent: Same Shape + Same Size

If you stamp a shape twice, the results are congruent. Every angle and every edge length matches perfectly.



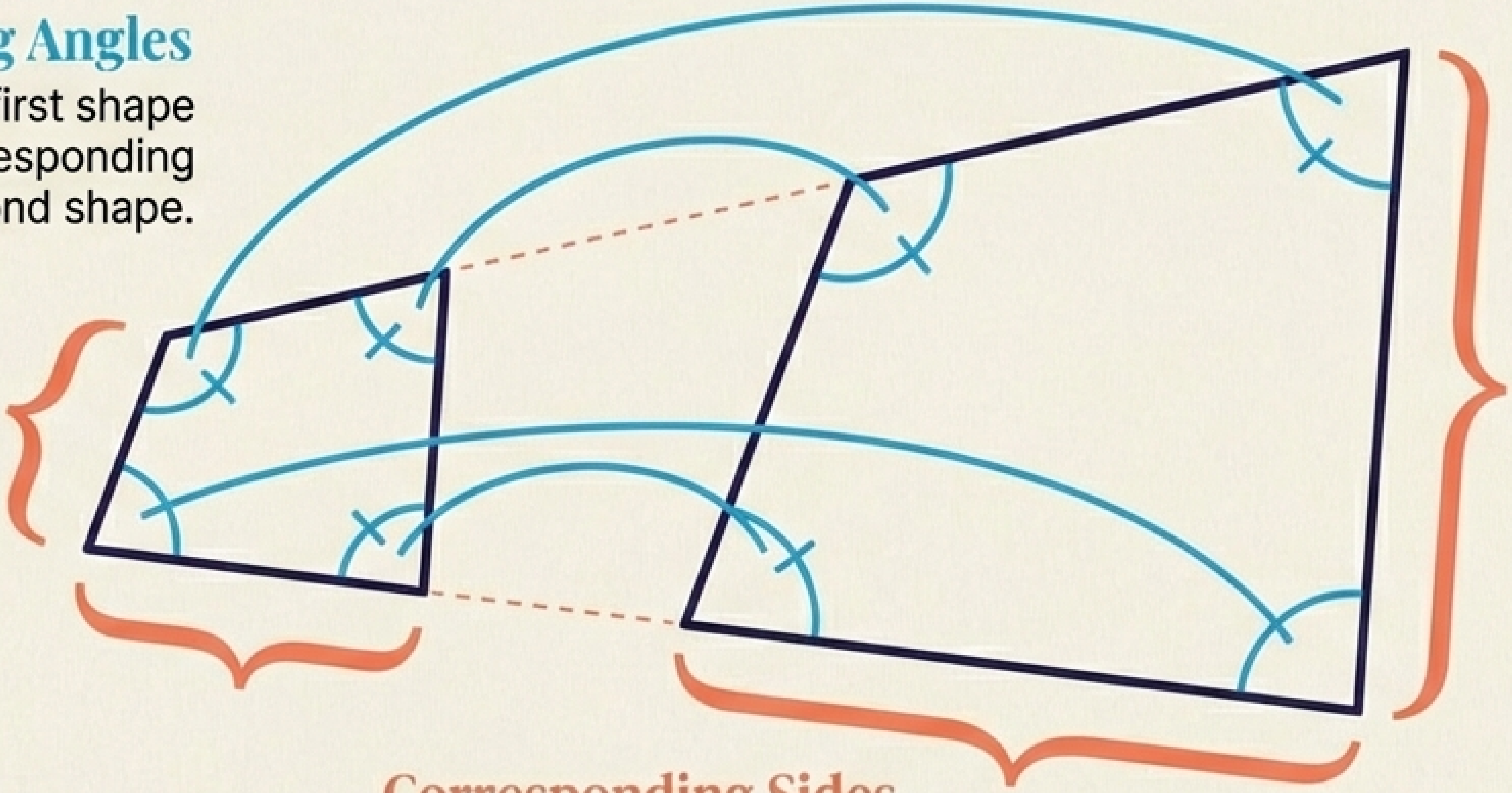
Similar: Same Shape + Different Size

If a photographer prints a negative into a passport photo and a large poster, the images are similar. The subjects haven’t distorted; every dimension has scaled by the exact same ratio.



The Two Rules of Similarity

For any two polygons with the same number of sides to be mathematically similar, they must satisfy two mandatory conditions simultaneously:



Corresponding Angles

Every angle in the first shape must equal its corresponding angle in the second shape.

Corresponding Sides

Every side length must scale by the exact same proportion (the scale factor).

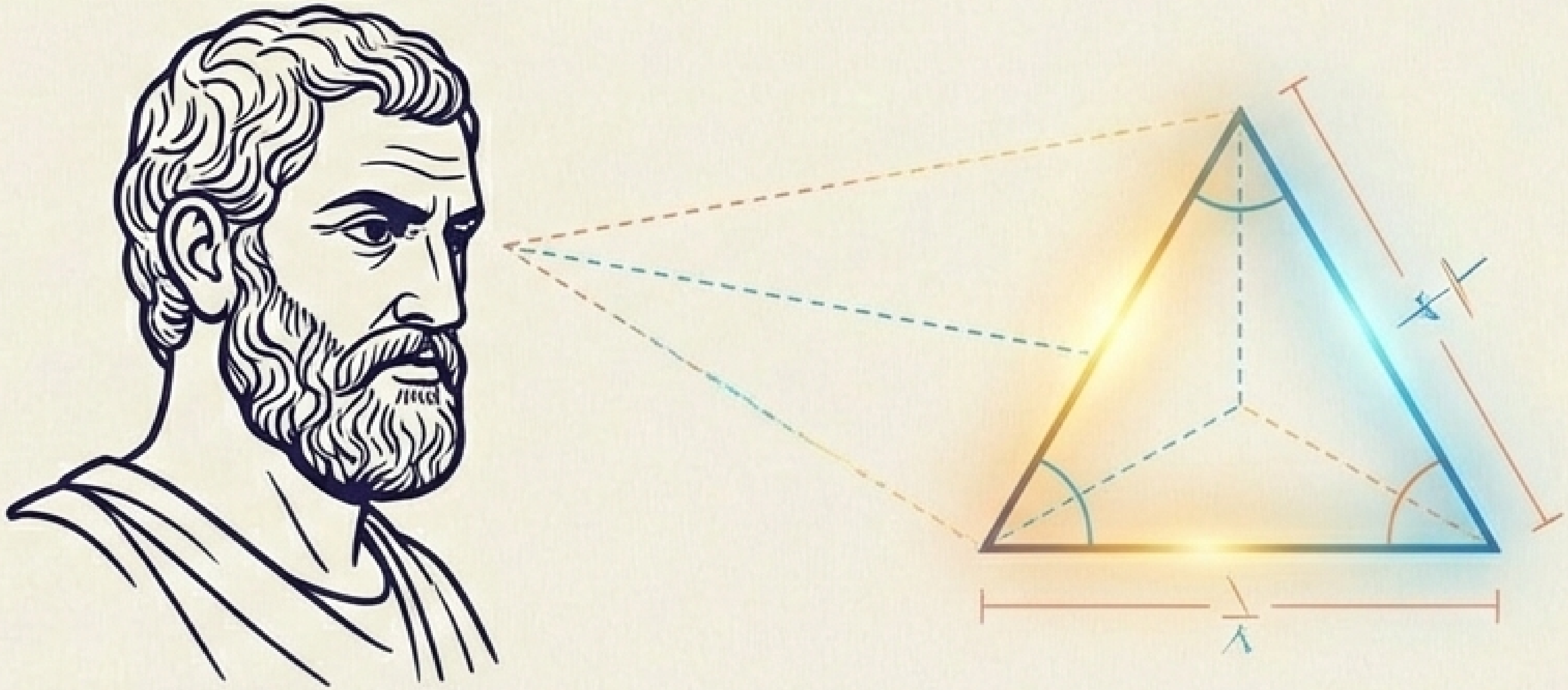
Corresponding Sides

Every side length must scale by the exact same proportion (the scale factor).

Enter the Triangle

Triangles are the foundational building blocks of all polygons. Because of their rigid nature, they have special properties when it comes to similarity.

If the corresponding angles of two triangles are equal, they are known as equiangular triangles. The ancient Greek mathematician Thales established a profound truth about them: the ratio of any two corresponding sides in two equiangular triangles is always exactly the same.

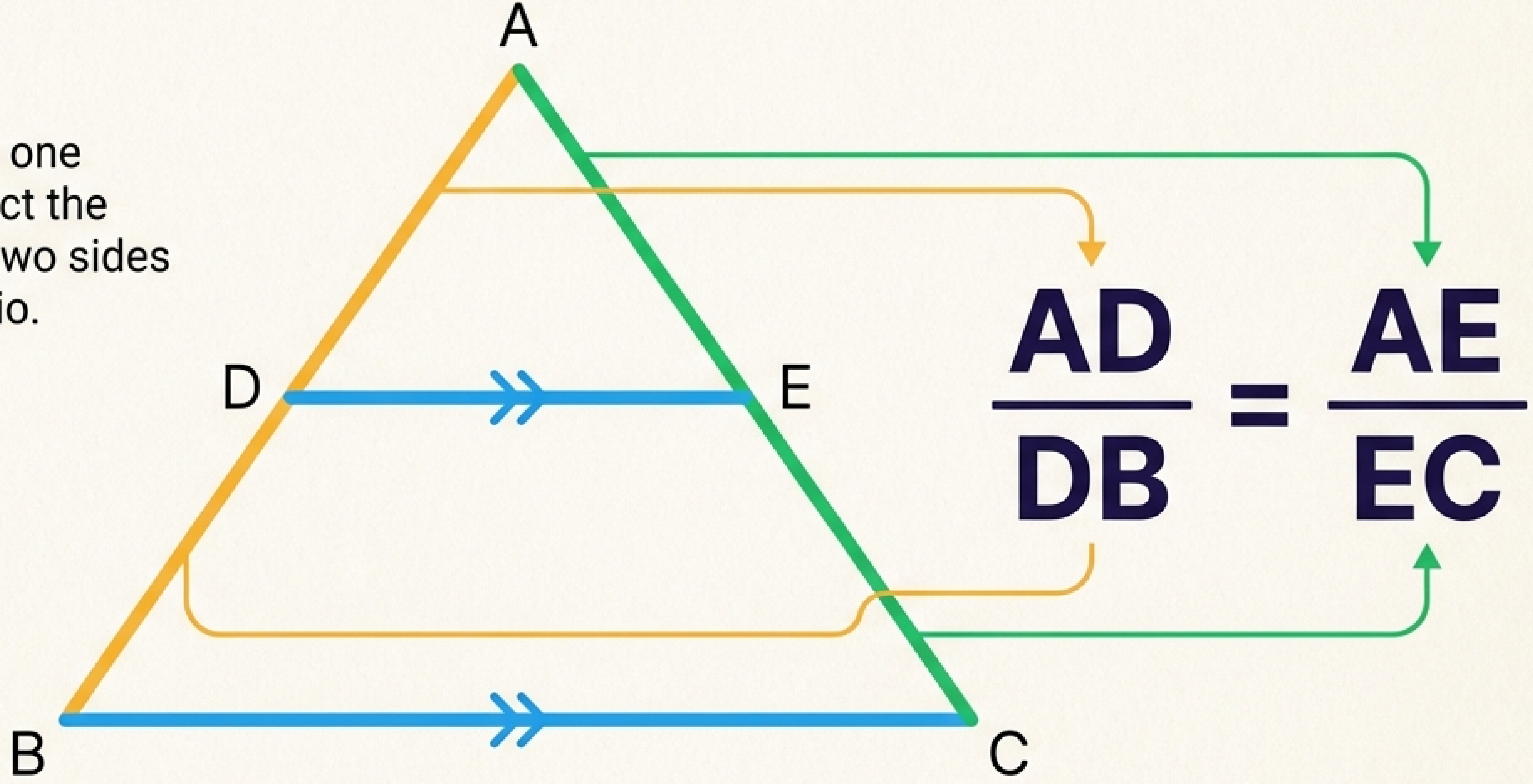


Tool 1: The Basic Proportionality Theorem

Also known as Thales' Theorem, this is the anchor of triangle similarity.

The Rule

If a line is drawn parallel to one side of a triangle to intersect the other two sides, the other two sides are divided in the same ratio.



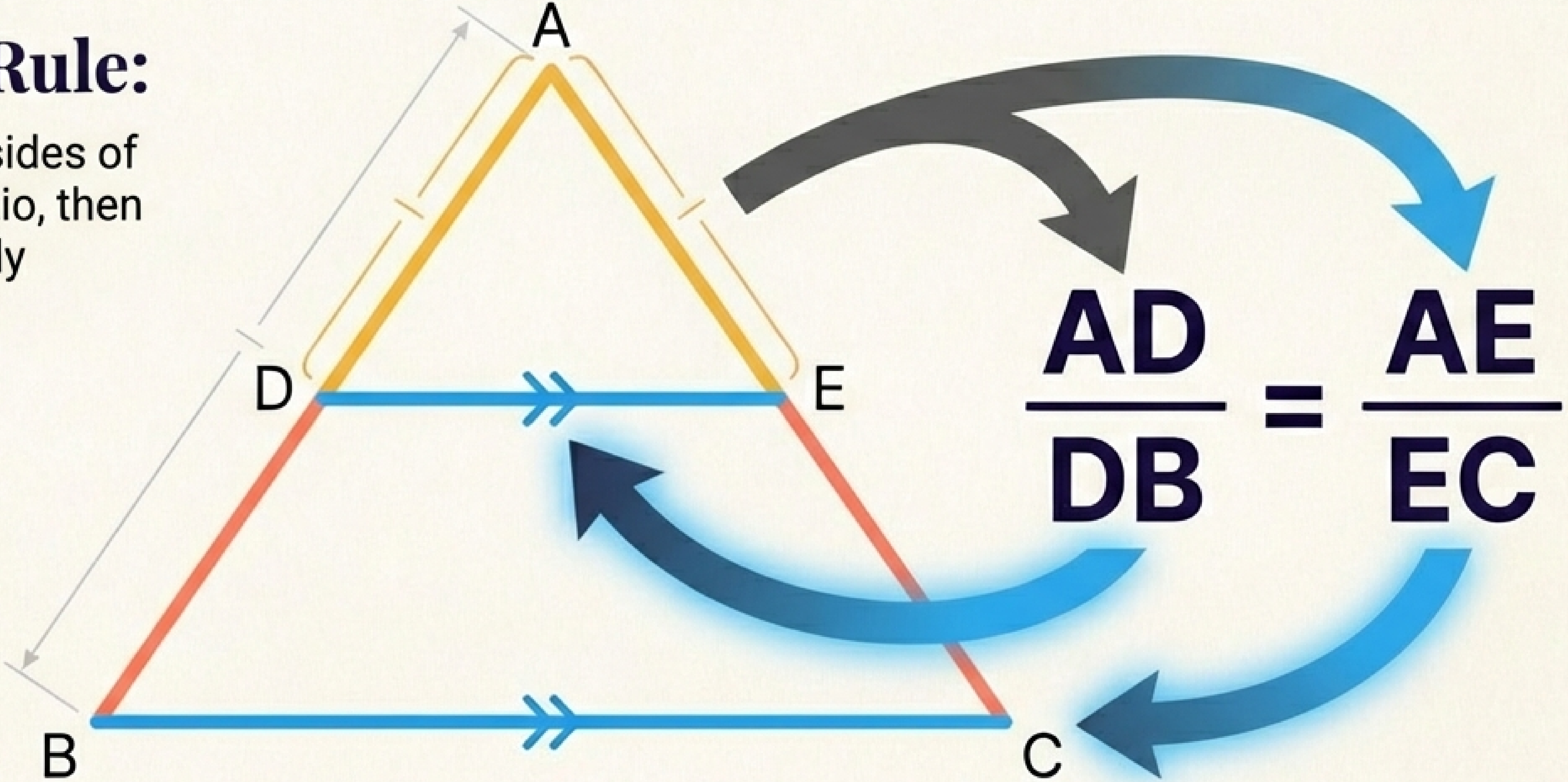
$$\frac{AD}{DB} = \frac{AE}{EC}$$

Reversing the Logic

The Basic Proportionality Theorem works in both directions.

The Converse Rule:

If a line divides any two sides of a triangle in the same ratio, then that line must be perfectly parallel to the third side.



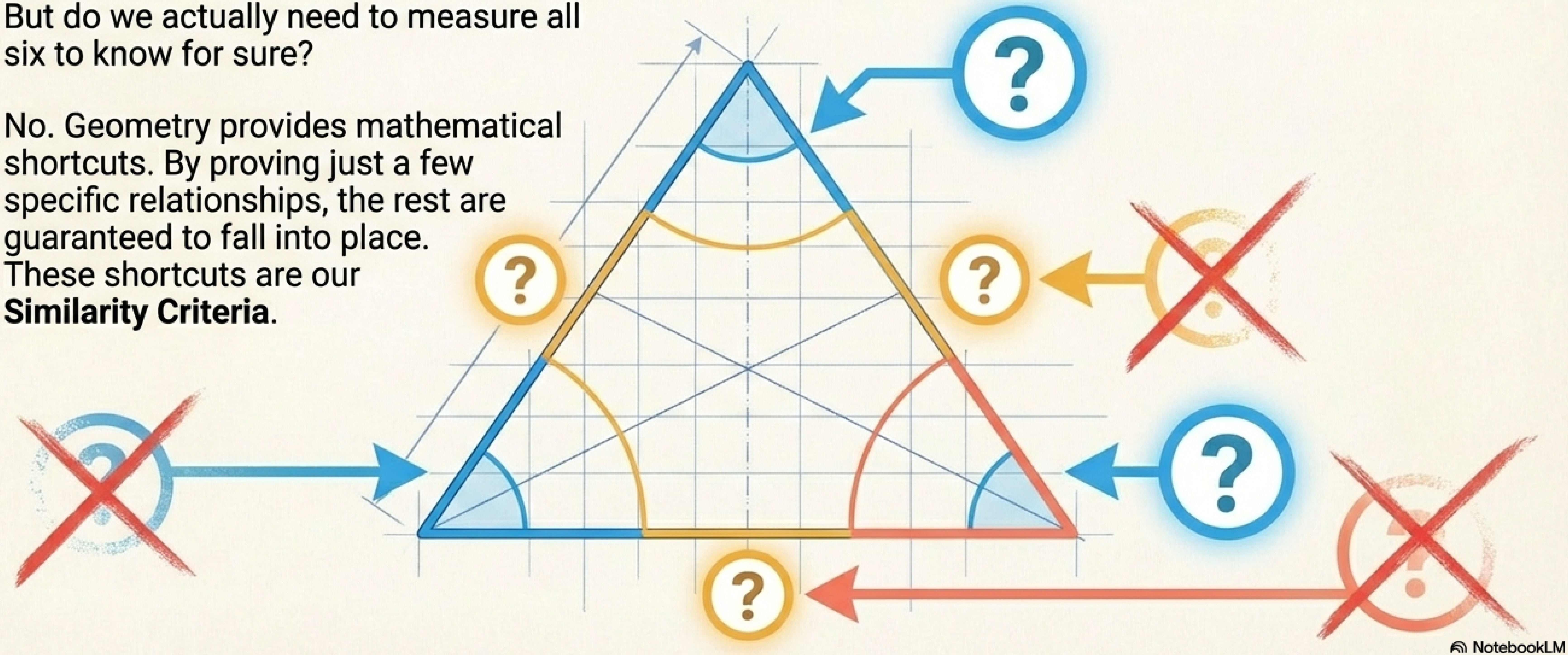
$$\frac{AD}{DB} = \frac{AE}{EC}$$

Do We Need to Measure Everything?

To prove two polygons are similar, we learned we must check everything: all corresponding angles AND all corresponding side ratios. For a triangle, that means measuring 6 specific conditions.

But do we actually need to measure all six to know for sure?

No. Geometry provides mathematical shortcuts. By proving just a few specific relationships, the rest are guaranteed to fall into place. These shortcuts are our **Similarity Criteria**.



Tool 2: The AAA Criterion

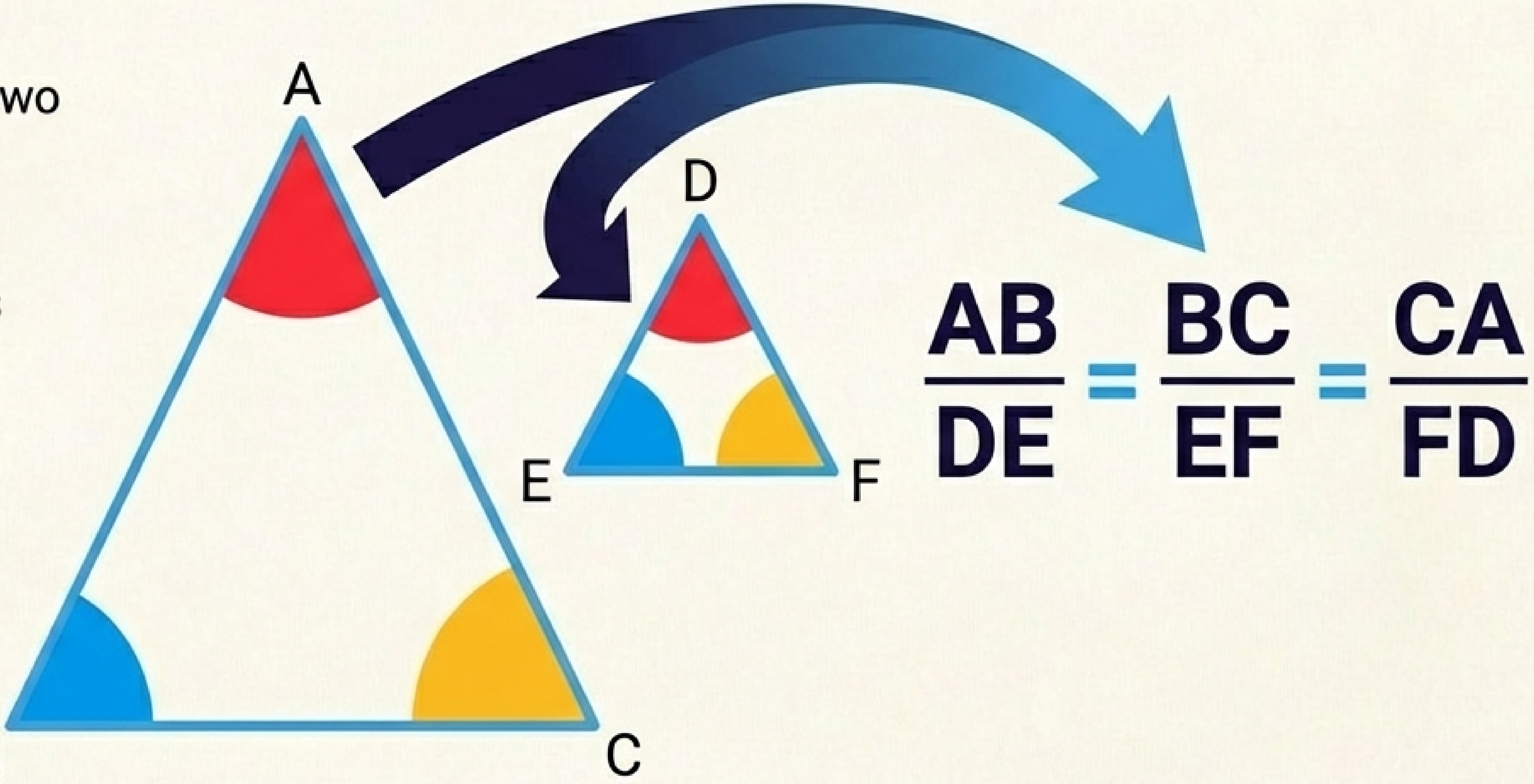
Angle-Angle-Angle Similarity

The Rule:

If corresponding angles in two triangles are equal, their corresponding sides are automatically proportional. Therefore, the two triangles are similar.

The AA Shortcut:

Because the interior angles of a triangle always add up to 180°, if just two angles match, the third must match perfectly as well.



$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$$

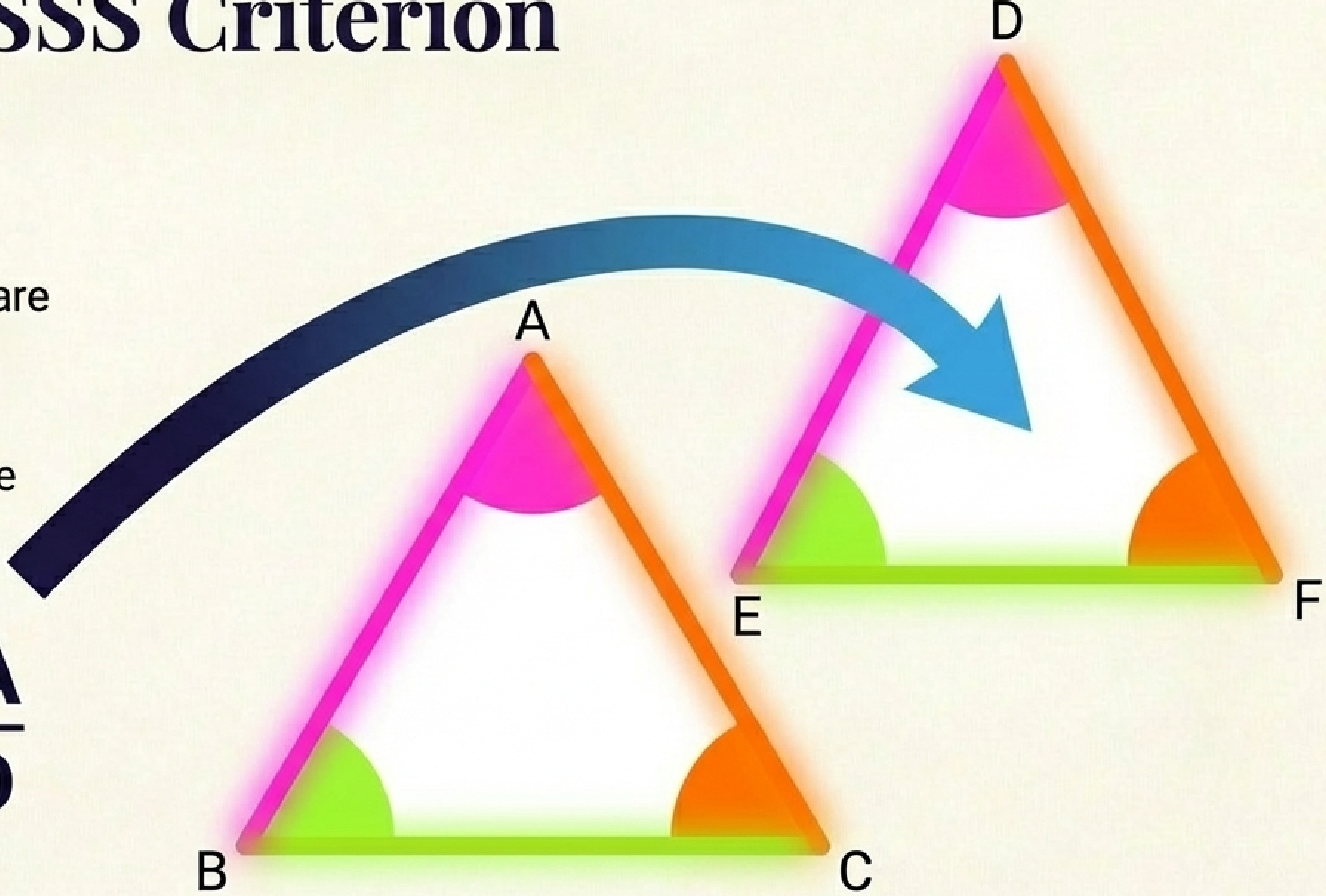
Tool 3: The SSS Criterion

Side-Side-Side Similarity

The Rule:

If the sides of one triangle are proportional to the sides of another triangle, then their corresponding angles are mathematically forced to be equal. Hence, the triangles are similar.

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$$



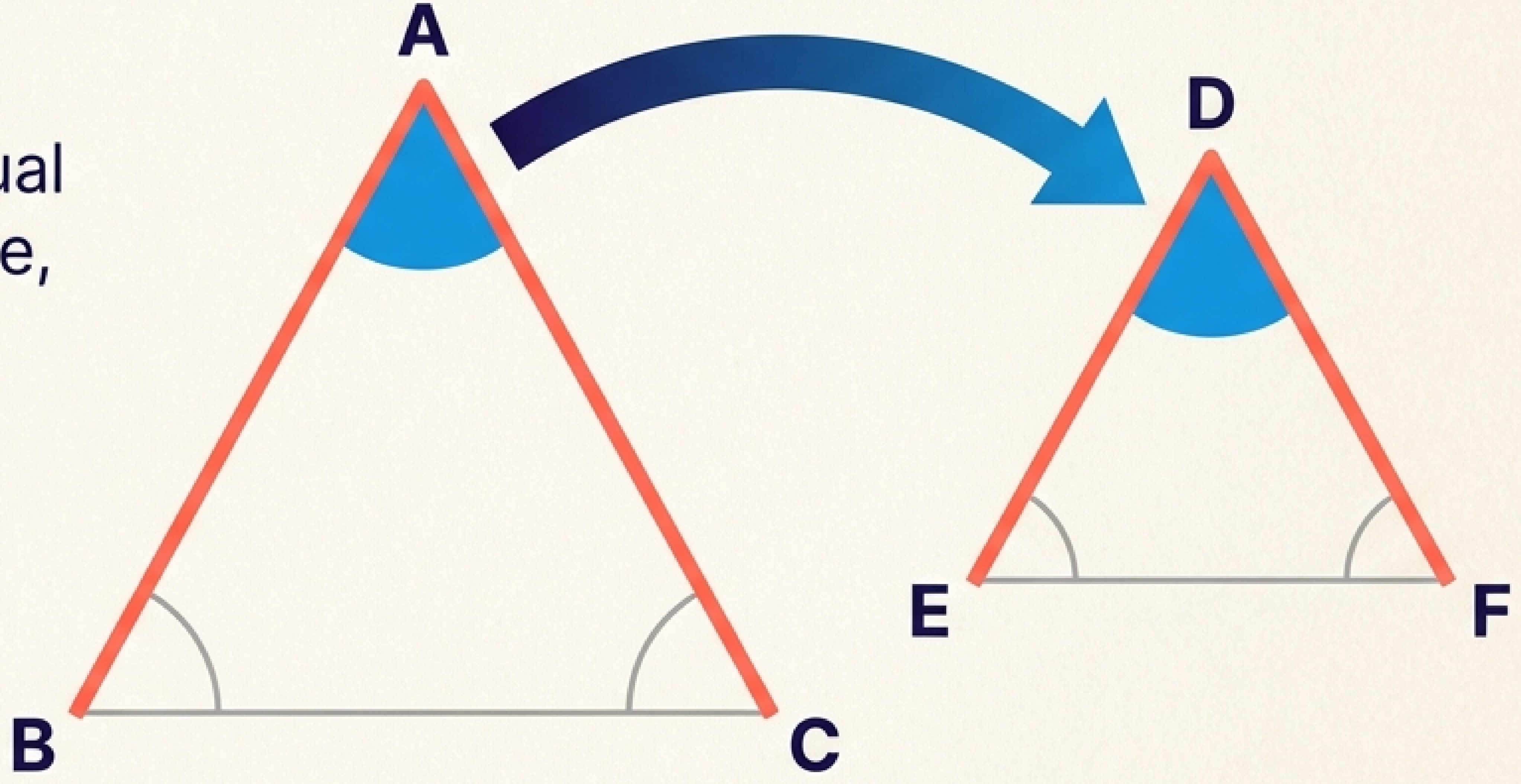
Tool 4: The SAS Criterion

Side-Angle-Side Similarity

The Rule:

If one angle of a triangle is equal to one angle of another triangle, and the two sides that include those angles are proportional, the two triangles are similar.

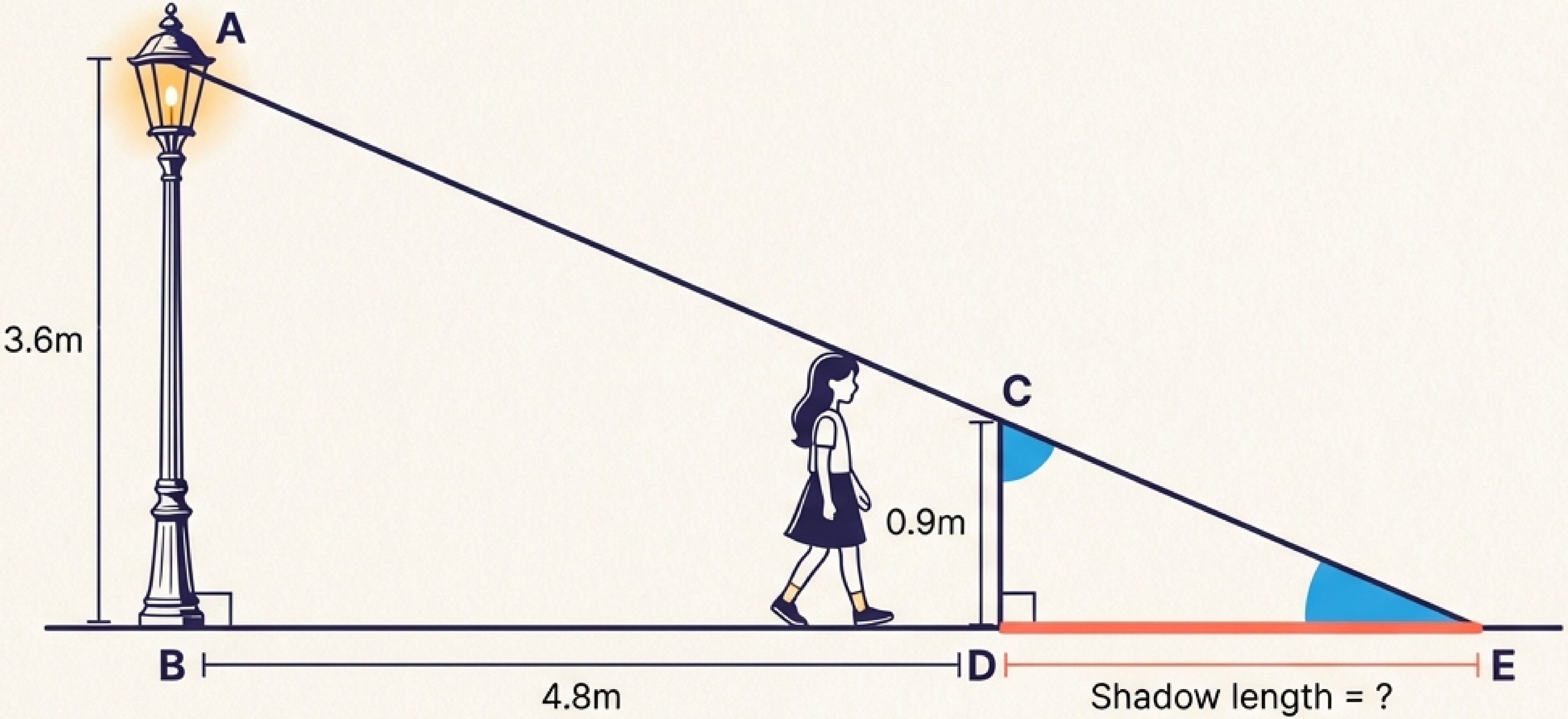
Angle A = Angle D AND
AB/DE = AC/DF



Application: The Moving Shadow

Let's put the toolkit to the test in the physical world.

A girl with a height of 90cm is walking away from the base of a 3.6m tall lamp-post at a speed of 1.2 meters per second. After exactly 4 seconds of walking, how long is the shadow she casts on the ground?

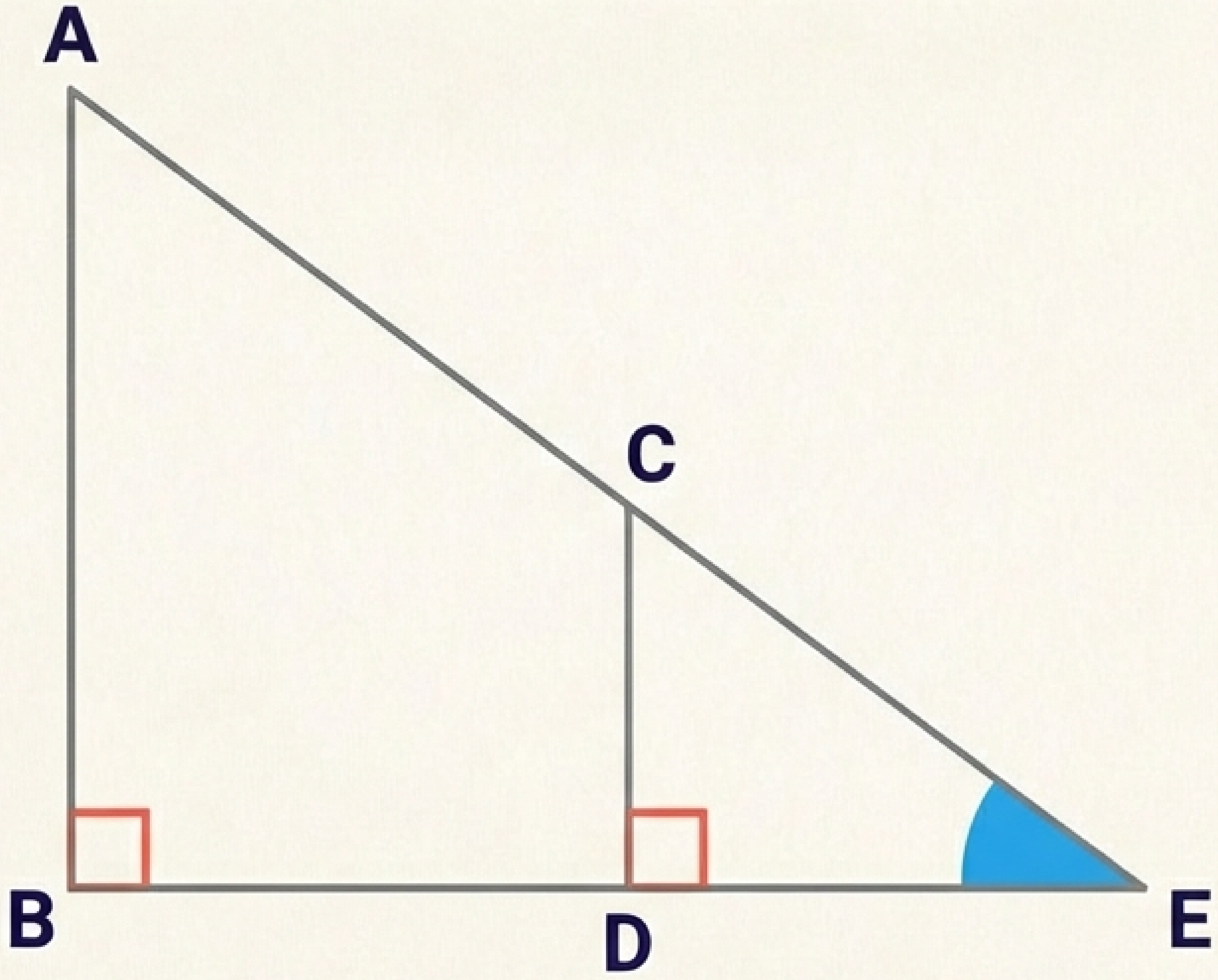


Structuring the Unmeasurable

First, we translate reality into geometry. We form two triangles: **Triangle ABE (the large triangle made by the lamp-post)** and **Triangle CDE** (the smaller triangle made by the girl).

1. The lamp and the girl are both standing perfectly vertical to the ground, giving both triangles a matching 90° angle (Angle B = Angle D).
2. Both triangles share the exact same point at the tip of the shadow (Angle E = Angle E).

By the AA Similarity Criterion, Triangle ABE is perfectly similar to Triangle CDE.



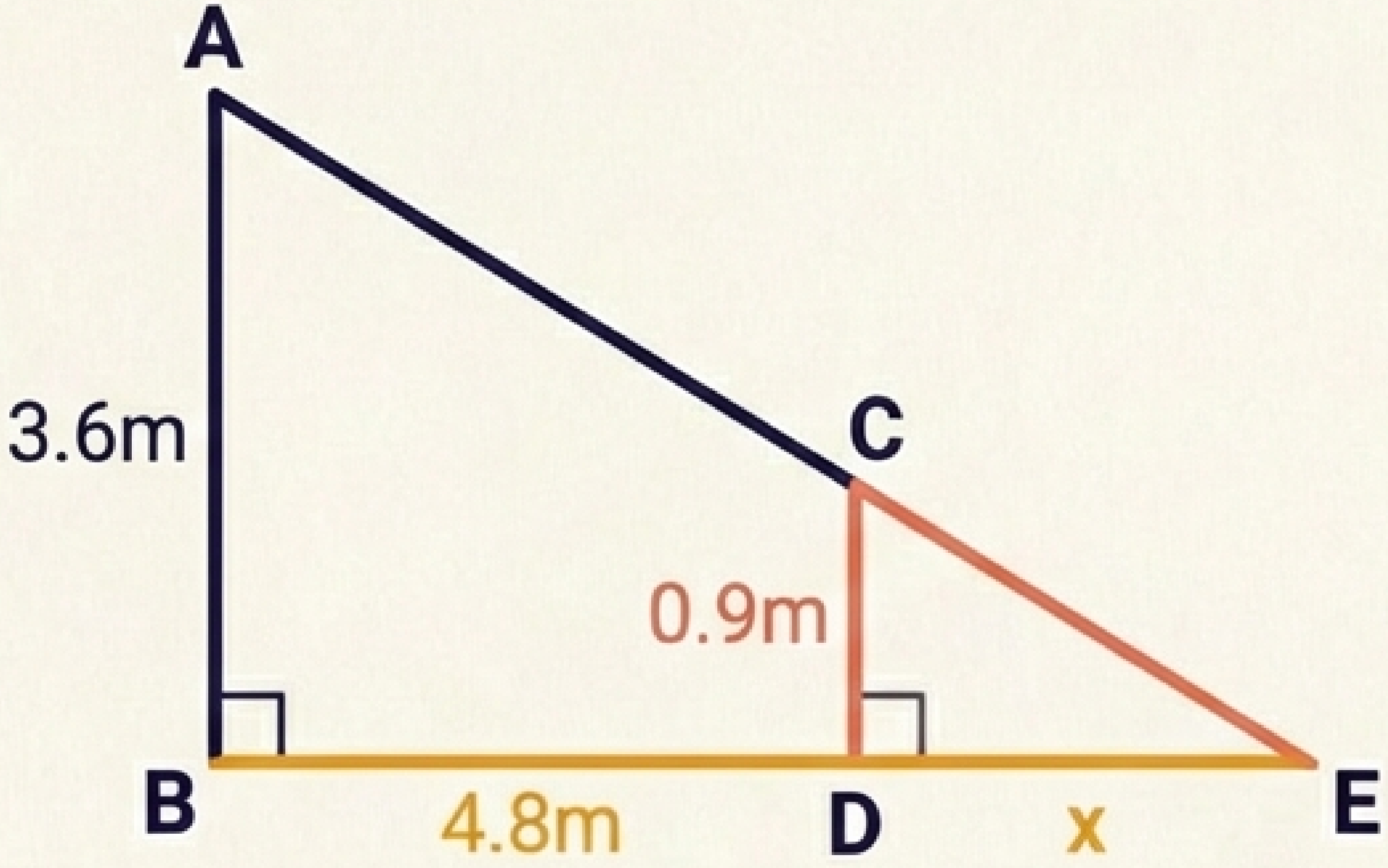
The Mathematics of the Shadow

Because the triangles are similar, their sides must be perfectly proportional.

$$\frac{BE}{DE} = \frac{AB}{CD}$$

We know her speed (1.2 m/s) and time (4s), so her distance from the lamp is 4.8m. Let the unknown shadow length (DE) be x.

The Calculation:



$$\frac{4.8 + x}{x} = \frac{3.6\text{m}}{0.9\text{m}}$$

$$4.8 + x = 4x$$

$$3x = 4.8$$

$$x = 1.6\text{m}$$

The length of the shadow is exactly 1.6 meters.

The Power of Proportions

From ancient philosophers estimating the height of pyramids using the sun, to modern applications in physics and engineering, the geometry of similarity proves that we don't need a tape measure to understand scale.

By understanding the proportional rules of the universe—the Basic Proportionality Theorem, AAA, SSS, and SAS—the unmeasurable heights and distances of our world become nothing more than simple arithmetic.