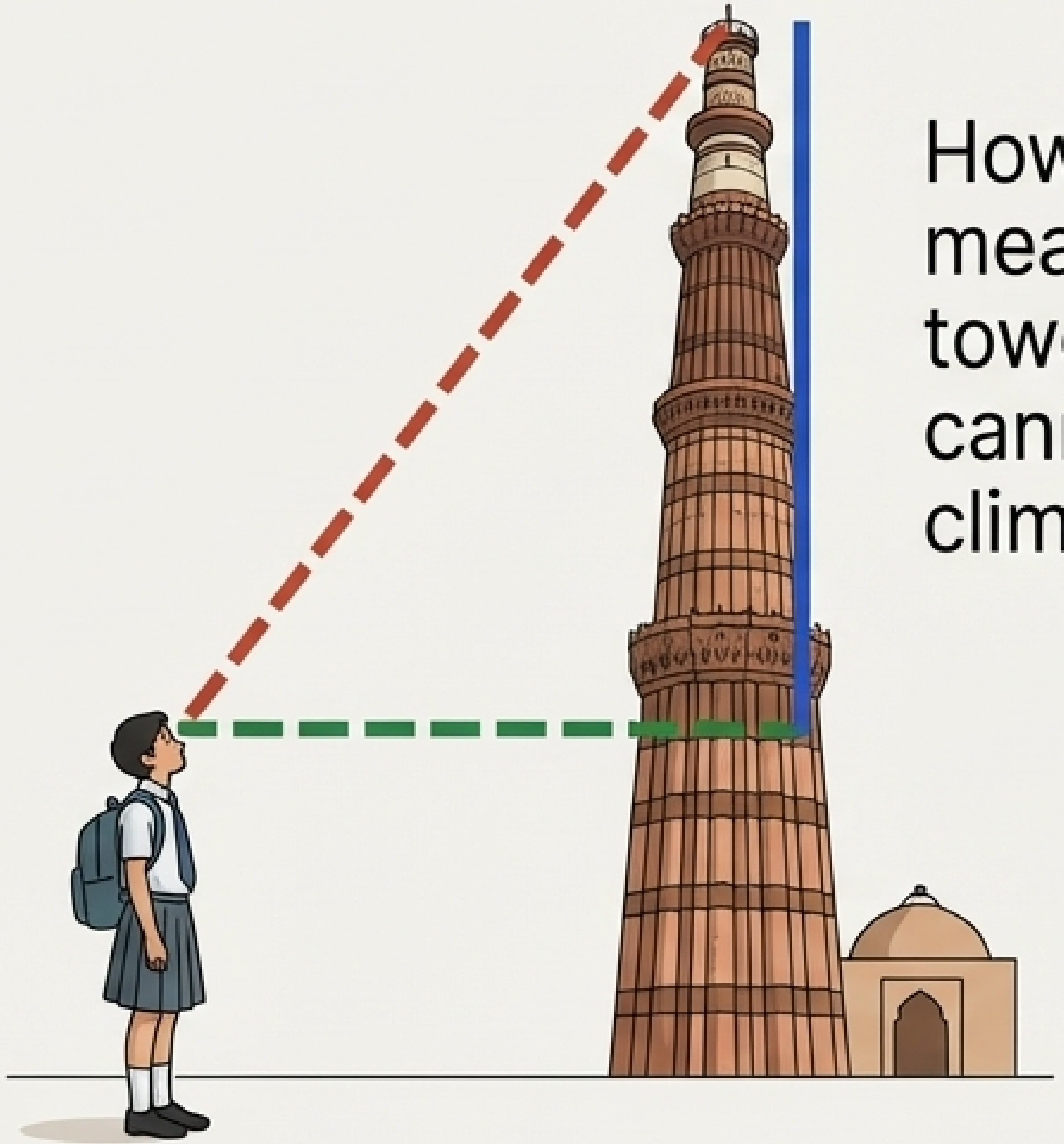
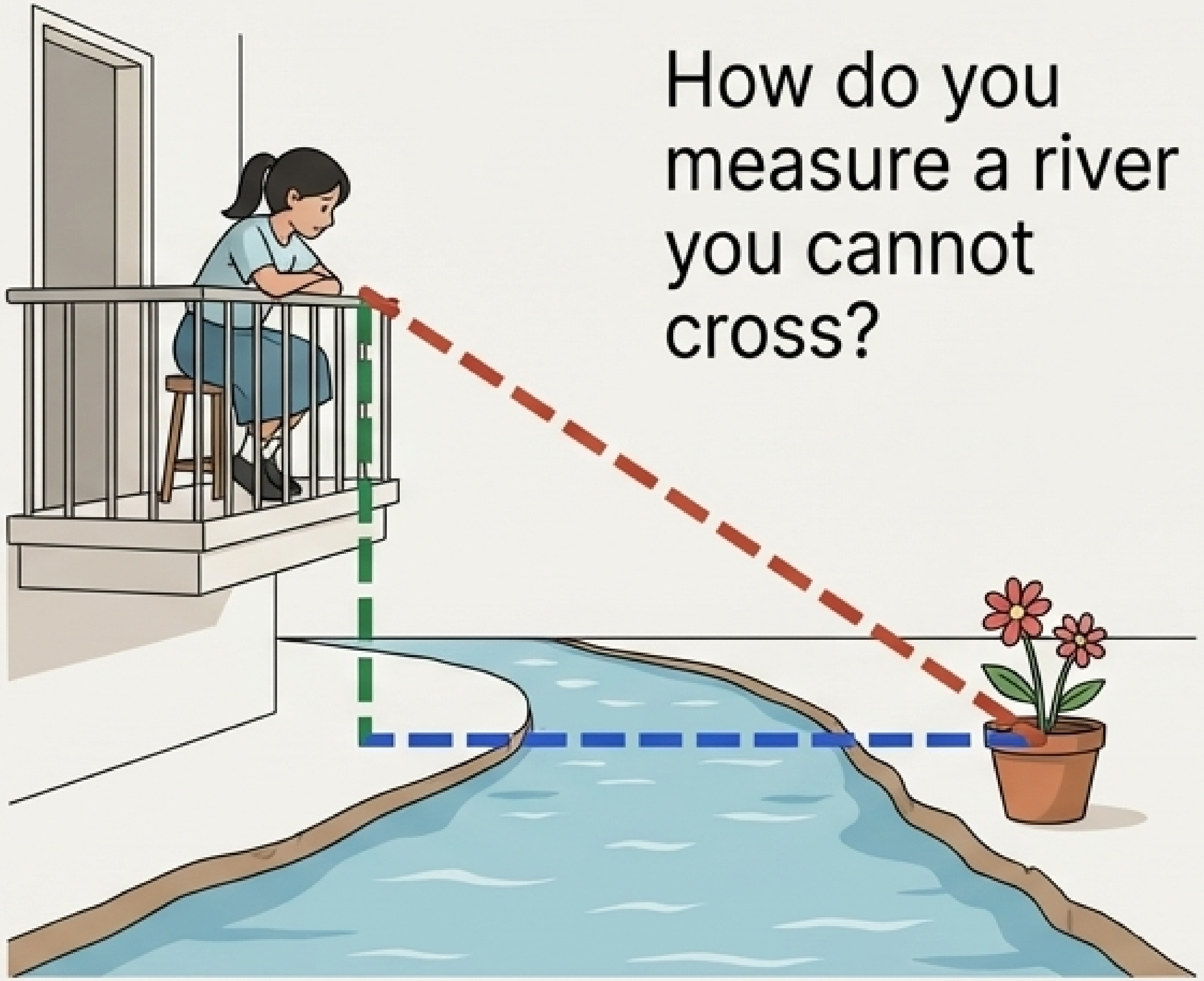
Measuring the Impossible



How do you measure a tower you cannot climb?



How do you measure a river you cannot cross?

By mapping physical reality to mathematical rules, we can unlock impossible distances. This is the power of Trigonometry.

The Geometry of Three Sides

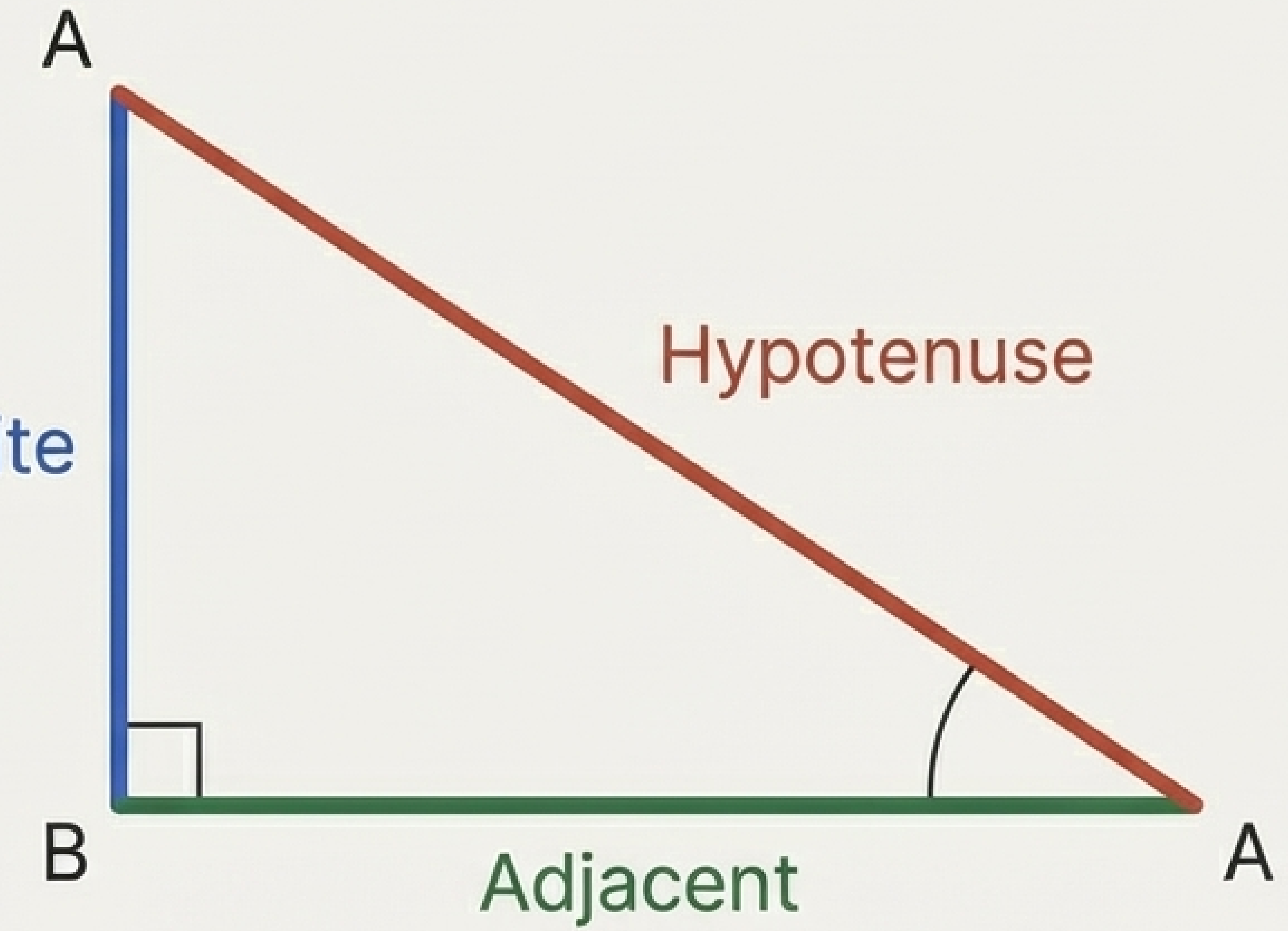
tri } (meaning three)
gon } (meaning sides)
metron } (meaning measure)

OME
TRY

Trigonometry is the study of relationships between the sides and angles of a right triangle.

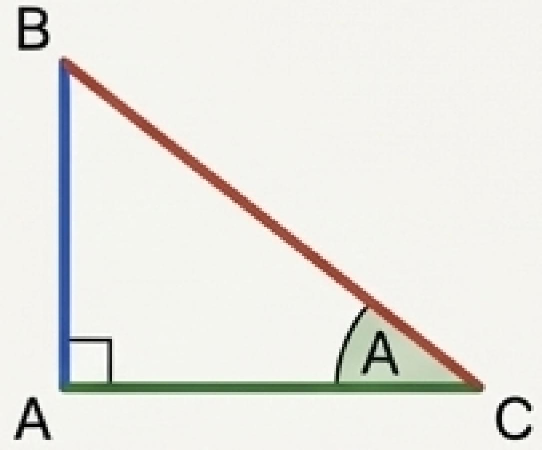
Tracing the Sine:
In 500 C.E., Indian mathematician Aryabhata introduced the concept of the half-chord, calling it ardha-jya, which was later shortened to jiva. Translated into Arabic, it became jiba, and eventually morphed into the Latin word sinus (meaning curve).
In 1626, Edmund Gunter abbreviated it to the modern sin.

The Anatomy of a Right Triangle



The position of the Opposite and Adjacent sides is entirely dependent on which acute angle you are referencing. They swap if we look from Angle C.

The Primary Ratios



$$\sin A \text{ (sine)} = \frac{\text{Opposite}}{\text{Hypotenuse}}$$

$$\cos A \text{ (cosine)} = \frac{\text{Adjacent}}{\text{Hypotenuse}}$$

$$\tan A \text{ (tangent)} = \frac{\text{Opposite}}{\text{Adjacent}}$$

Notice that $\tan A = \frac{\sin A}{\cos A}$.

Important: “sin A” is a single mathematical abbreviation. It is not a multiplication of “sin” and “A”.

The Reciprocal Ratios

cosec A (cosecant) = 1

$\sin A = \frac{\text{Hypotenuse}}{\text{Opposite}}$

sec A (secant) = 1

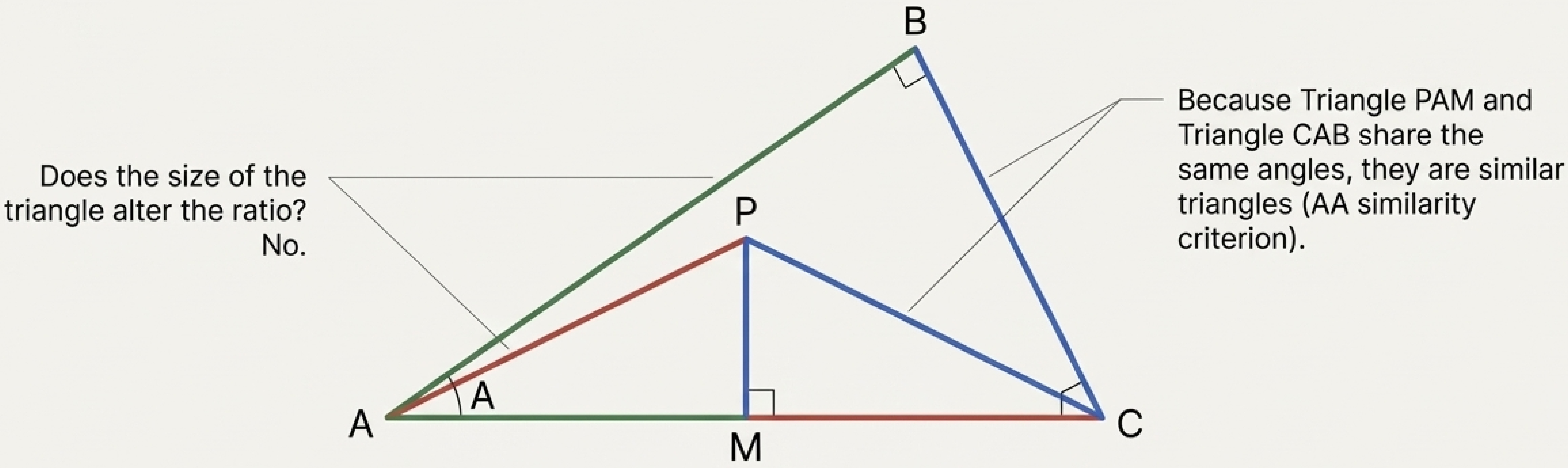
$\cos A = \frac{\text{Hypotenuse}}{\text{Adjacent}}$

cot A (cotangent) = 1

$\tan A = \frac{\text{Adjacent}}{\text{Opposite}}$

Because the Hypotenuse is always the longest side, the value of sin A or cos A is always less than or equal to 1. Conversely, sec A and cosec A are always greater than or equal to 1.

The Law of Invariance: Proportion is King



Therefore, their sides are strictly proportional:

$$\frac{MP}{AP} = \frac{BC}{AC} = \sin A$$

The values of the trigonometric ratios do not vary with the lengths of the sides, as long as the angle remains the same.

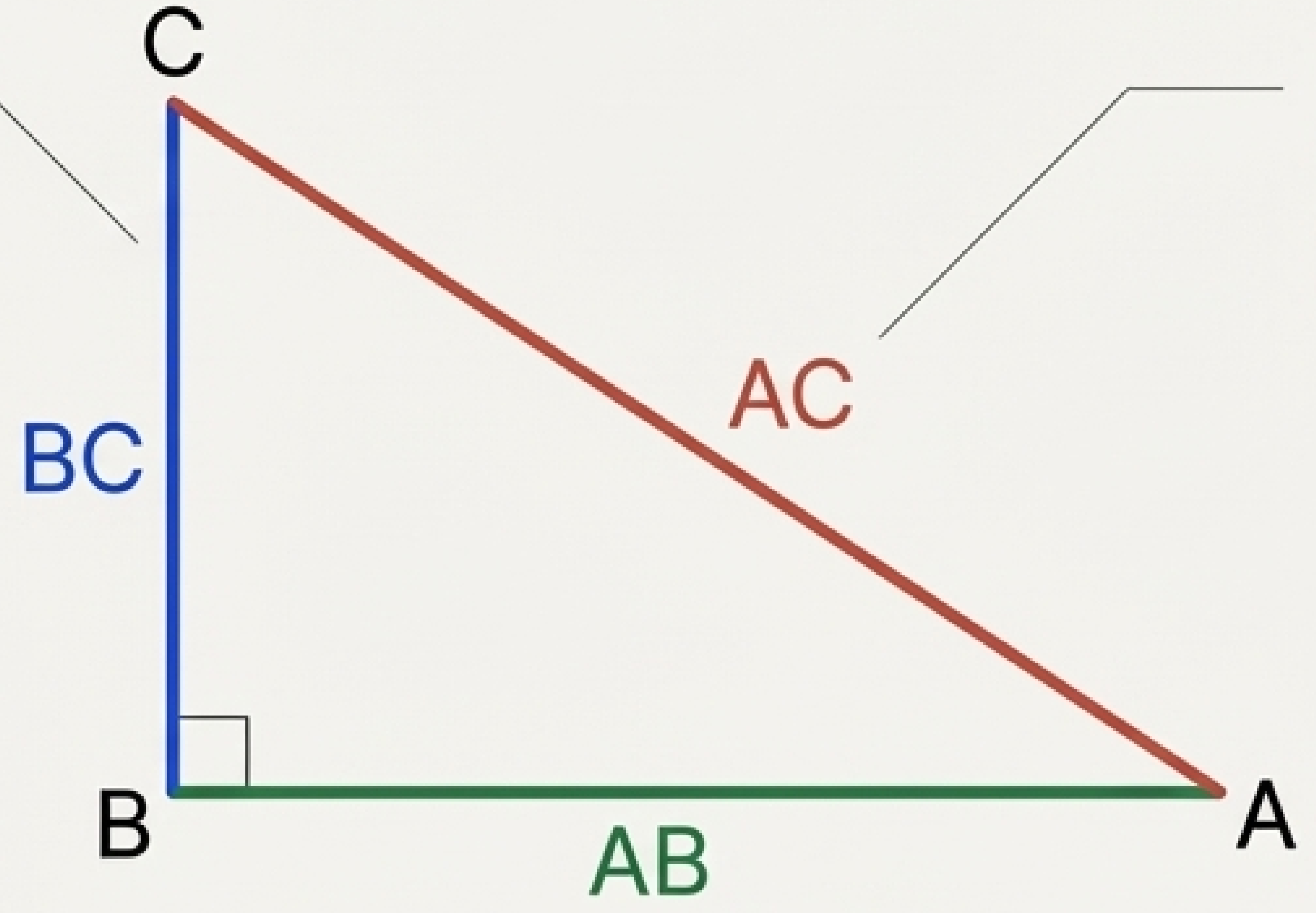
Standard Angle Matrix

	0°	30°	45°	60°	90°
sin A	0	$1/2$	$1/\sqrt{2}$	$\sqrt{3}/2$	1
cos A	1	$\sqrt{3}/2$	$1/\sqrt{2}$	$1/2$	0
tan A	0	$1/\sqrt{3}$	1	$\sqrt{3}$	Not defined
cosec A	Not defined	2	$\sqrt{2}$	$2/\sqrt{3}$	1
sec A	1	$2/\sqrt{3}$	$\sqrt{2}$	2	Not defined
cot A	Not defined	$\sqrt{3}$	1	$1/\sqrt{3}$	0

Observe the boundary limits: As Angle A increases from 0° to 90°, sin A grows from 0 to 1, while cos A shrinks from 1 to 0.

The Master Key: Pythagoras

A Trigonometric Identity is an equation involving ratios that remains true for all values of the angle involved.



Every core identity in trigonometry is born from one geometric law: The Pythagorean Theorem.

$$AB^2 + BC^2 = AC^2$$

By dividing this single equation by the square of each of its sides, we unlock the universal rules of trigonometry.

Deriving the Core Identities

(Divide by AC^2)

$$\frac{AB^2}{AC^2} + \frac{BC^2}{AC^2} = \frac{AC^2}{AC^2}$$

$$(\cos A)^2 + (\sin A)^2 = 1$$

Identity 1:

$$\cos^2 A + \sin^2 A = 1$$

(Divide by AB^2)

$$\frac{AB^2}{AB^2} + \frac{BC^2}{AB^2} = \frac{AC^2}{AB^2}$$

$$1 + (\tan A)^2 = (\sec A)^2$$

Identity 2:

$$1 + \tan^2 A = \sec^2 A$$

(Divide by BC^2)

$$\frac{AB^2}{BC^2} + \frac{BC^2}{BC^2} = \frac{AC^2}{BC^2}$$

$$(\cot A)^2 + 1 = (\operatorname{cosec} A)^2$$

Identity 3:

$$\cot^2 A + 1 = \operatorname{cosec}^2 A$$

The Middle Position of Mathematics

$$\sin^2 A + \cos^2 A = 1$$

With just one known ratio, or one measured side and an acute angle, the entire geometric reality of a physical space can be decoded.

“There is perhaps nothing which so occupies the middle position of mathematics as trigonometry.” – J.F. Herbart (1890)