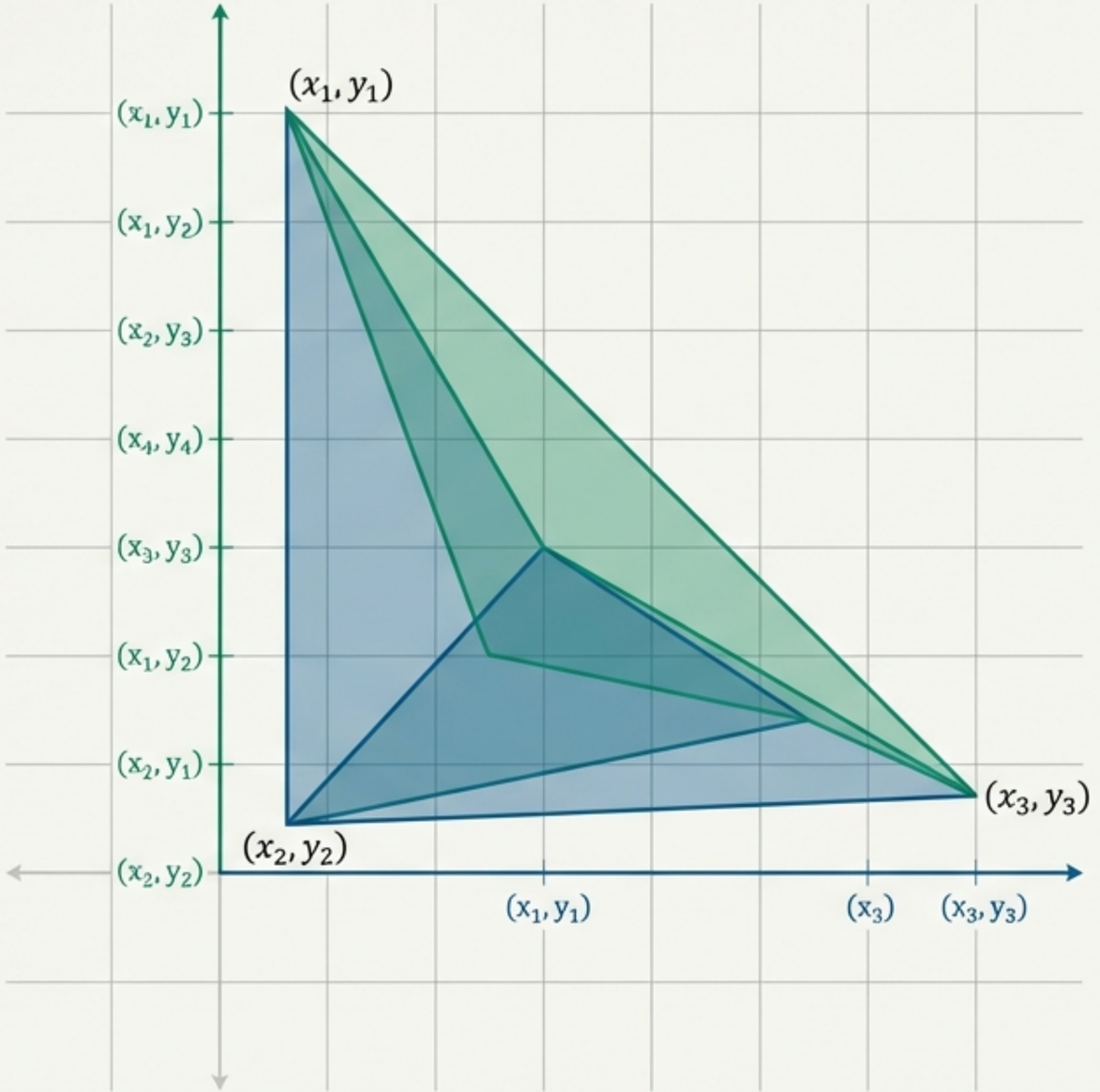
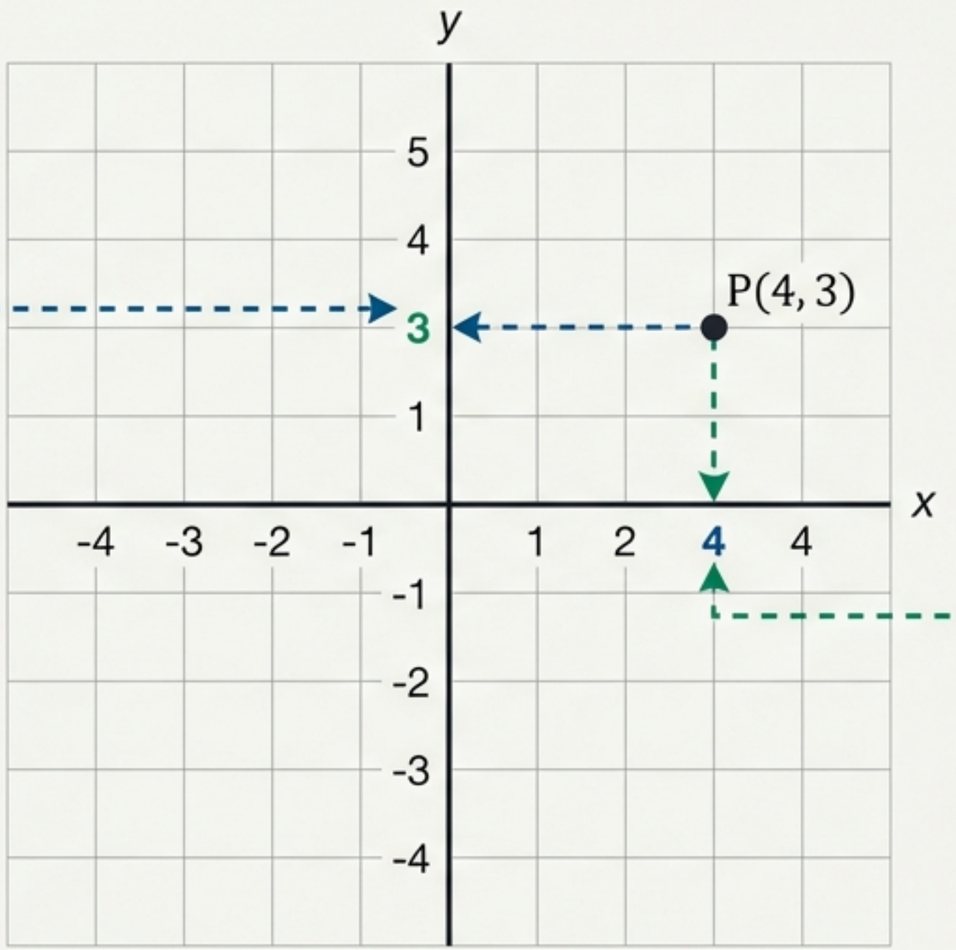
Coordinate Geometry

The Bridge Between Algebra and Physical Space



Translating space into numbers. Used in physics, engineering, and navigation.

The Foundation of the Plane

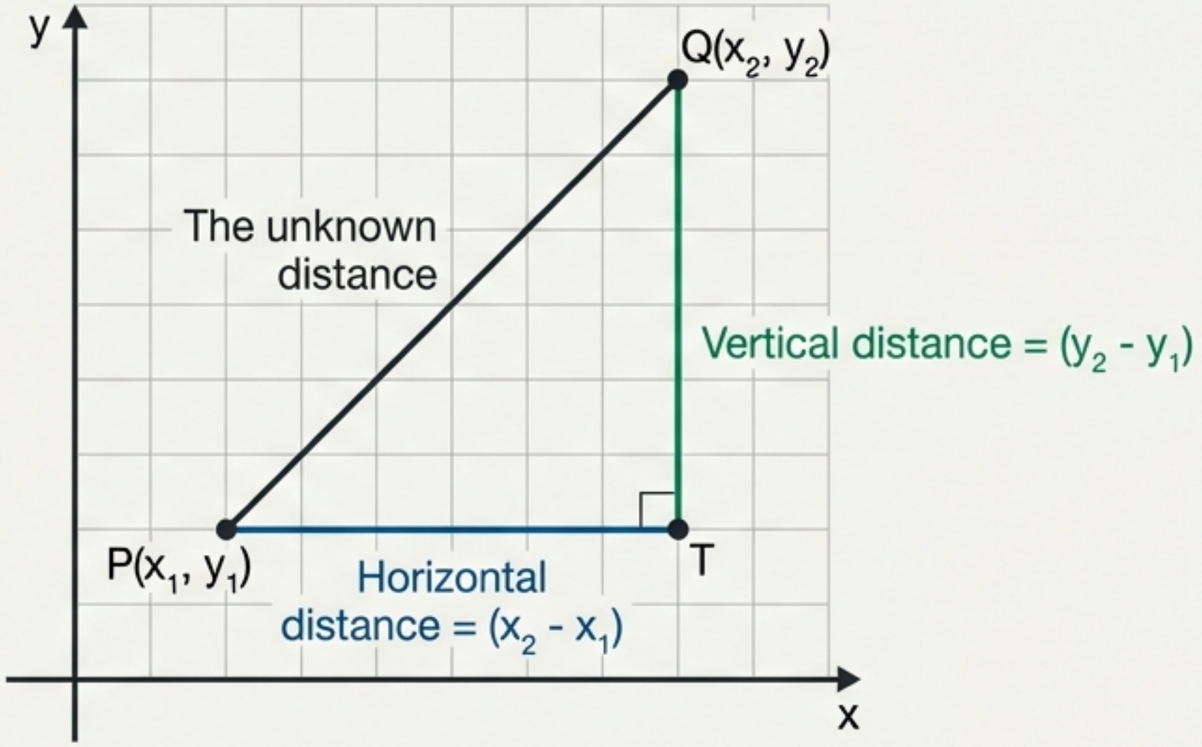


x-coordinate (Abscissa) -
Distance from the y-axis.

y-coordinate (Ordinate) -
Distance from the x-axis.

A point **P(x, y)** represents an exact location in 2D space.

The Distance Problem: A Visual Proof



By applying Pythagoras ($a^2 + b^2 = c^2$) to the right triangle PTQ, we get:

$$PQ^2 = PT^2 + QT^2.$$

The Distance Formula

$$PQ = \sqrt{((x_2 - x_1))^2 + (y_2 - y_1))^2}$$

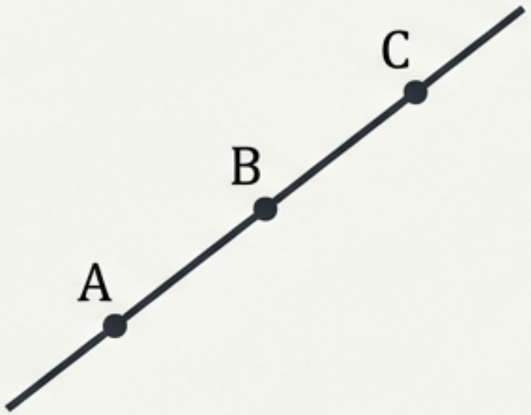
Special Case: Distance from the Origin

$$OP = \sqrt{(x^2 + y^2)}$$

When measuring from (0,0), the formula simplifies.

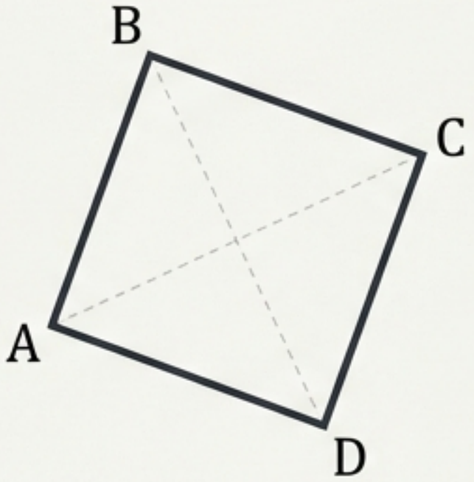
Applications of the Distance Formula

Collinearity



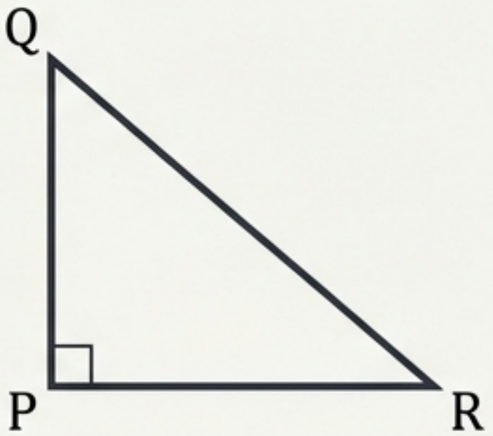
If AB + BC = AC, the points form a straight line.

Identifying Squares



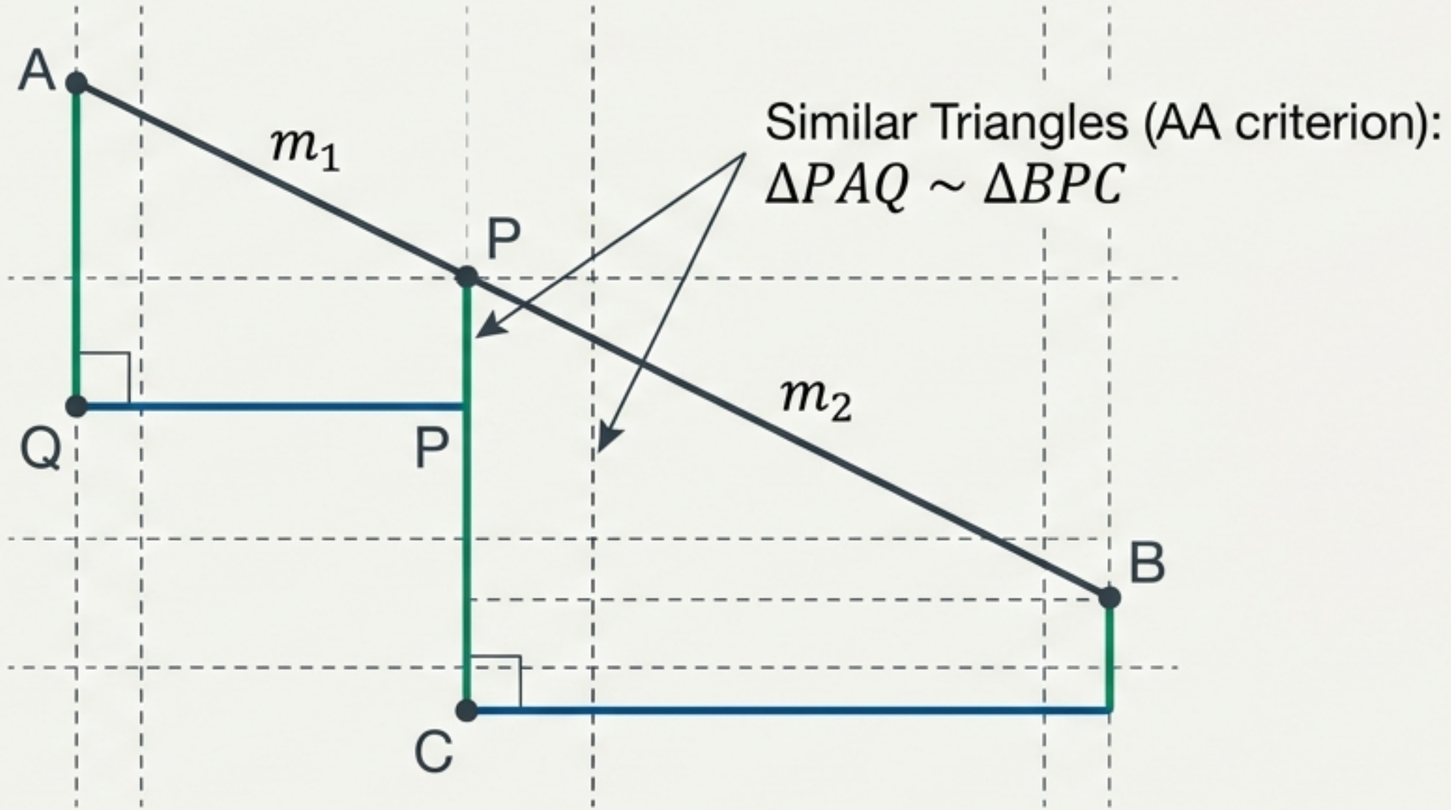
All 4 sides equal (AB=BC=CD=DA) AND both diagonals equal (AC=BD).

Right Triangles



Converse of Pythagoras: If $PQ^2 + PR^2 = QR^2$, angle P is 90°.

The Division Problem: A Visual Proof



Because the triangles are similar, their sides are proportional: *PA/BP = AQ/PC = PQ/BC*.

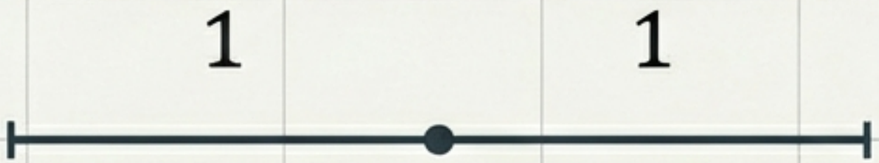
The Section Formula

$$P(x, y) = \left(\frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}, \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2} \right)$$

Finds the exact coordinates of point P dividing segment AB internally in the ratio $m_1 : m_2$.

Special Cases & Shortcuts

The Mid-point Formula



$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

When $m_1 = m_2$, simply average the coordinates.

Finding Unknown Ratios

$$\left(\frac{kx_2 + x_1}{k+1}, \frac{ky_2 + y_1}{k+1}\right)$$

When solving for a missing ratio, substitute m_1:m_2 with k:1 to reduce the equation to a single unknown variable.

The Coordinate Geometry Toolkit

Distance

$$\mathrm{PQ} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Distance from Origin

$$\mathrm{OP} = \sqrt{x^2 + y^2}$$

Section Formula

$$\mathrm{P}(x, y) = \left(\frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}, \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}\right)$$

Mid-point

$$\mathrm{P}(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$