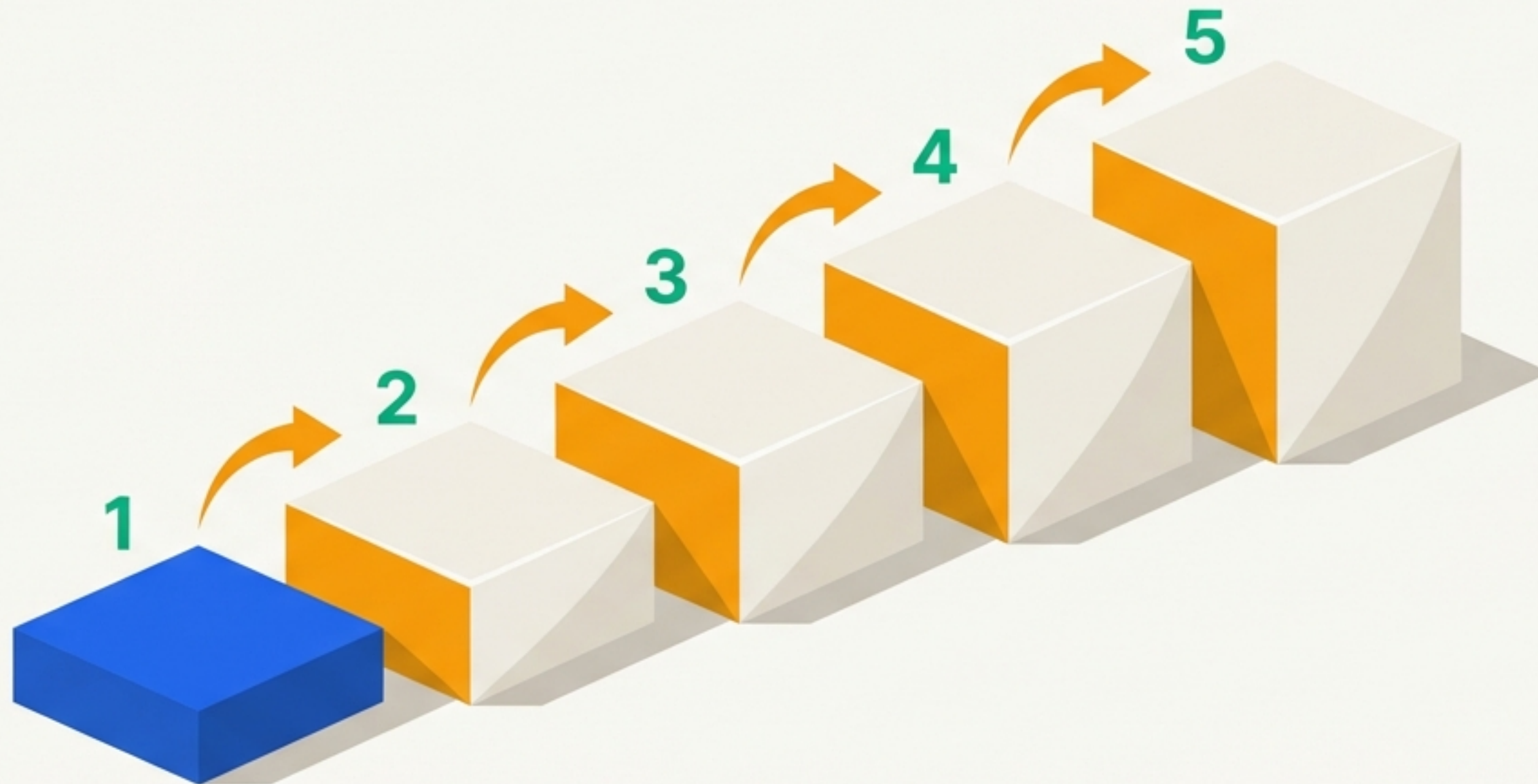


Decoding the Rhythm of Numbers

A Visual Guide to Arithmetic Progressions



Spotting the Pattern

Reena's Starting Salary



8000 → 8500 → 9000 → 9500

Adds **500** every year.

Rungs on a Ladder

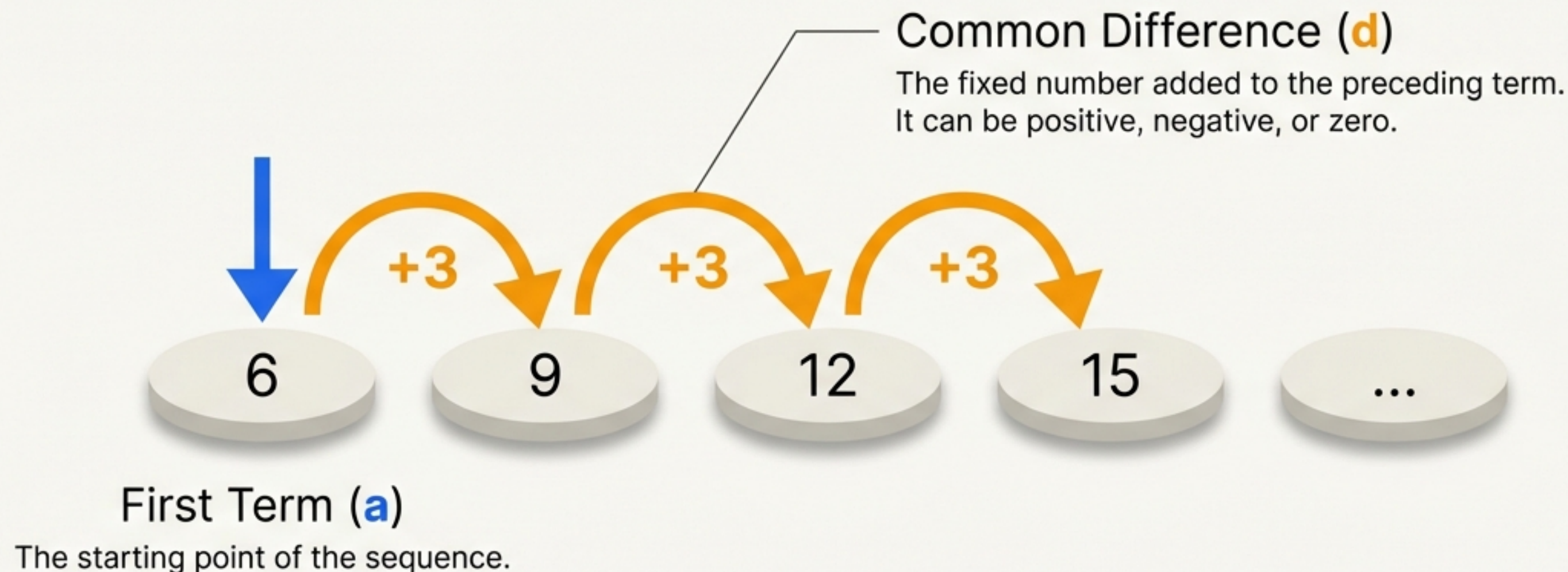


45cm → 43cm → 41cm → 39cm

Subtracts **2cm** every step.

Whether growing or shrinking, both patterns progress by a fixed, predictable step.

The Anatomy of a Progression



In general, an AP looks like this: $a, a + d, a + 2d, a + 3d...$

Finite vs. Infinite Journeys



Has a definitive end point.

147, 148, 149 ... 157 ← Last Term (l)

1, 2, 3, 4, 5... 2, 3, 4, 5, 6, 7, 8, 9, 10

Continues forever. No last term.

Tool #1: The n th Term Formula

The diagram shows the formula $a_n = a + (n - 1)d$ with arrows pointing to each part and its meaning:

- a_n : The destination you want to find.
- a : Your starting point.
- n : The number of jumps. (Why -1 ? Because you don't jump to land on the first term!)
- d : The size of each jump.

Solution Box

Reena's 25th-year salary is:

$$8000 + (25 - 1) \times 500 = 20,000$$

Problem solved instantly.

The Boy Genius and the Summation Problem

At age 10, Carl Friedrich Gauss was told to sum the numbers from 1 to 100. Instead of adding them one by one, he found a geometric trick.



He realized there were exactly 50 pairs of 101.

$$50 \times 101 = 5050$$

This brilliant trick of 'folding' the sequence birthed our second powerful tool.

Tool #2: Calculating the Total Sum (**S**)

The Gauss Shortcut

Use when you know the Last Term (**l**).

$$S = \left(\frac{n}{2}\right) (a + l)$$

The number of pairs.

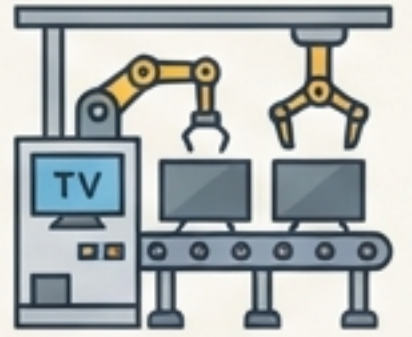
The value of each folded pair.

The General Formula

Use when you only know **a**, **d**, and **n**.

$$S = \left(\frac{n}{2}\right) [2a + (n - 1)d]$$

This is just Box 1, but replacing **l** with our Tool #1 formula!



Formulas in Action: The Factory Problem

A manufacturer produces 600 TVs in Year 3 ($a_3 = 600$), and 700 TVs in Year 7 ($a_7 = 700$). Production increases by a fixed number yearly.

Step 1: Find the DNA

Using Tool 1, we solve the equations to find:

$a = 550$ (Year 1 production)

$d = 25$ (yearly increase)

Step 2: The Target

What is the total production over 7 years?

Step 3: Tool 2

$$S_7 = \frac{7}{2} [2(550) + (7 - 1)25] = 4375 \text{ TVs.}$$

Variable replament

Yearly increase

Takeaway: With just two data points, we decoded the entire past, present, and future of the factory.

The Arithmetic Progression Cheat Sheet

The Variables

a = First Term

d = Common Difference

n = Number of terms

l = Last term

The Core Rule

$$d = a_{k+1} - a_k$$

The difference between any consecutive terms is always constant.

Finding Any Term

$$a_n = a + (n - 1)d$$

Finding the Sum

$$S_n = \frac{n}{2} (a + l)$$
$$S_n = \frac{n}{2} [2a + (n - 1)d]$$