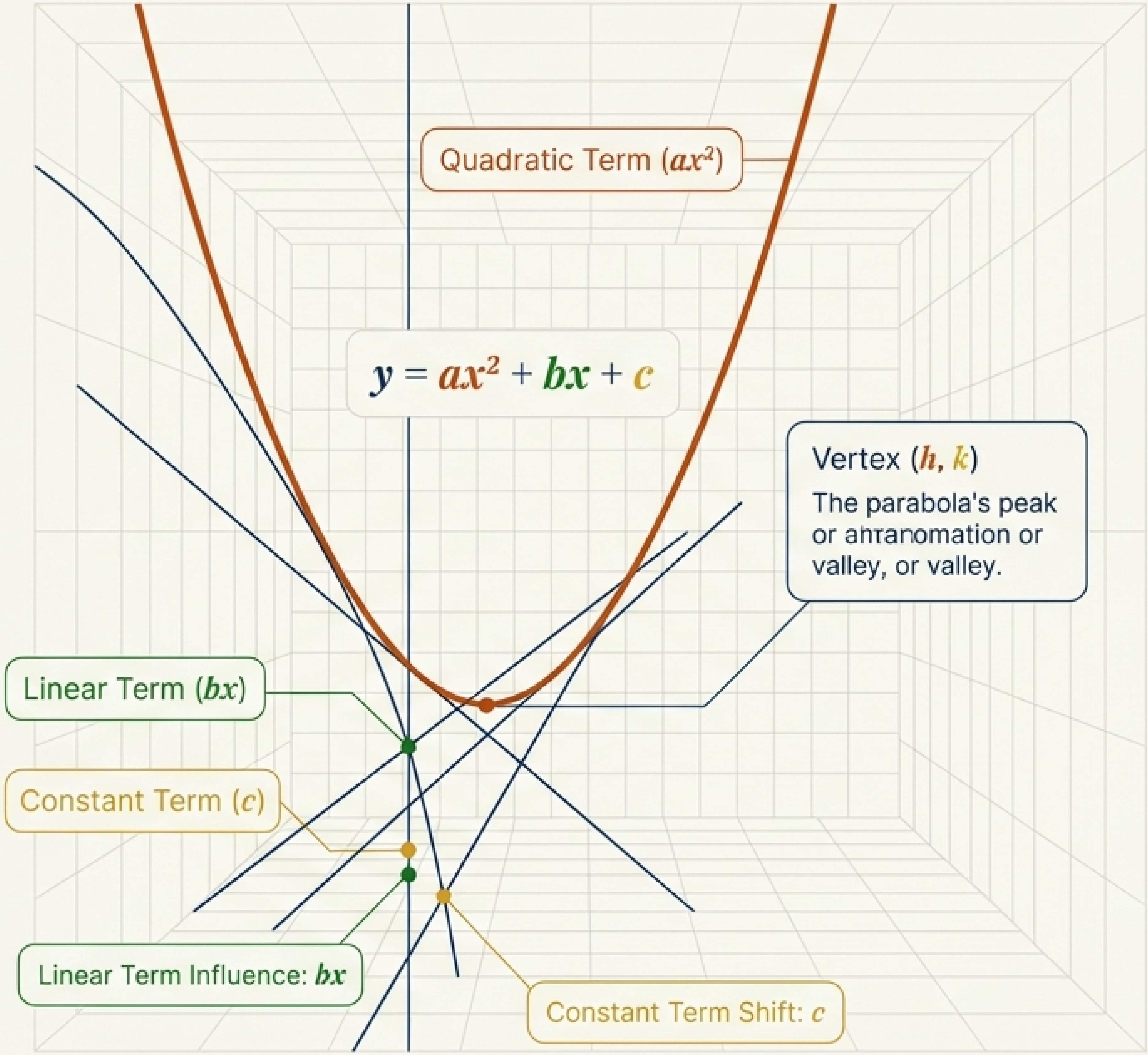
Quadratic Equations: The Mathematics of Space and Curves

Unlocking real-world dimensions through algebra.



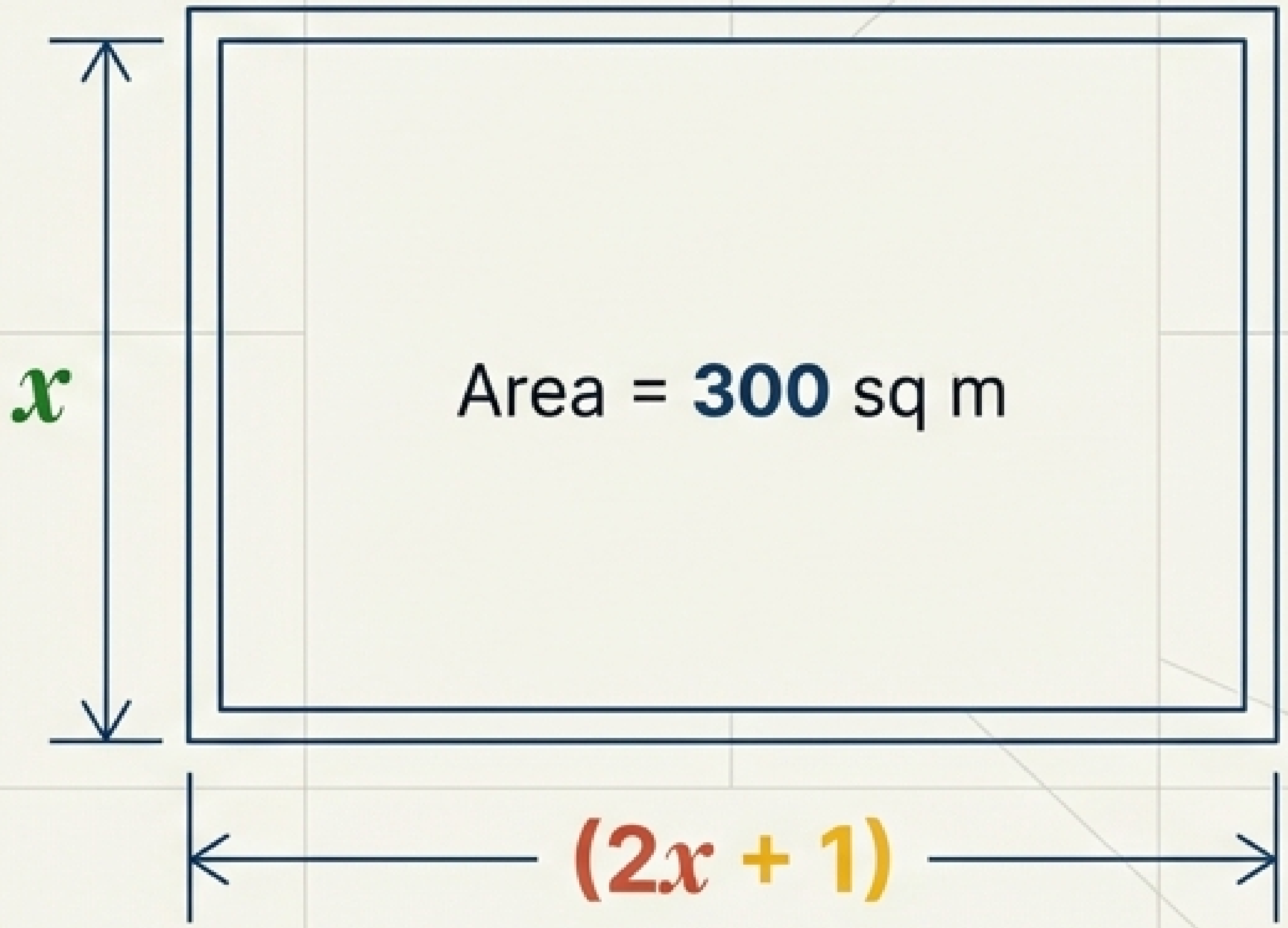
NotebookLM

The Charity Prayayer Hall Dilemma

A charity trust wants to build a prayer hall. They have two strict requirements:

1. The exact carpet area must be 300 square meters.
2. The length of the hall must be exactly 1 meter more than twice its breadth.

How do we determine the precise dimensions to give to the builders?



Translating Reality into Math

We know the formula for the area of a rectangle is Length × Breadth.
Let's plug in what we know.

$$Area = Length \times Breadth$$

$$\downarrow$$

$$300 = (2x + 1) \cdot x$$

$$\downarrow$$

$$300 = 2x^2 + x$$

$$\downarrow$$

$$2x^2 + x - 300 = 0$$

To find the breadth (x), we must solve this specific equation.

The Anatomy of a Quadratic Equation

Any equation describing a polynomial of degree 2 is a quadratic equation. When organized in descending order, we get the standard form.

$$ax^2 + bx + c = 0$$

- ax^2: Quadratic Term (where $a \neq 0$)
- bx: Linear Term
- c: Constant Term

Note: If $a = 0$, the equation is no longer quadratic; it becomes a straight line.

Is it a Quadratic Equation? (The Disguise)

Equations often hide their true nature. Always simplify first.

Looks linear, is quadratic

$$(x - 2)^2 + 1 = 2x - 3$$

↓ Expands to

$$x^2 - 4x + 5 = 2x - 3$$

↓ Simplifies to

$$x^2 - 6x + 8 = 0$$ ✓ Yes!

Looks quadratic, is linear

$$x(x + 1) + 8 = (x + 2)(x - 2)$$

↓ Expands to

$$x^2 + x + 8 = x^2 - 4$$

↓ Simplifies to

$$x + 12 = 0$$ ✗ No!

(The x^2 terms cancel out)

A Global Heritage of Problem Solving

Babylonians

Euclid
(Greece)

Brahmagupta
(India, 598 C.E.)

Al-Khwarizmi
(Arabia, ~800 C.E.)

Sridharacharya
(India, 1025 C.E.)

Babylonians

First to find two numbers with a given sum and product ($x^2 - px + q = 0$).

Euclid
(Greece)

Developed geometric approaches to finding these specific lengths.

Brahmagupta
(India, 598 C.E.)

Gave an explicit formula for solving equations of the form $ax^2 + bx = c$.

Al-Khwarizmi
(Arabia, ~800 C.E.)

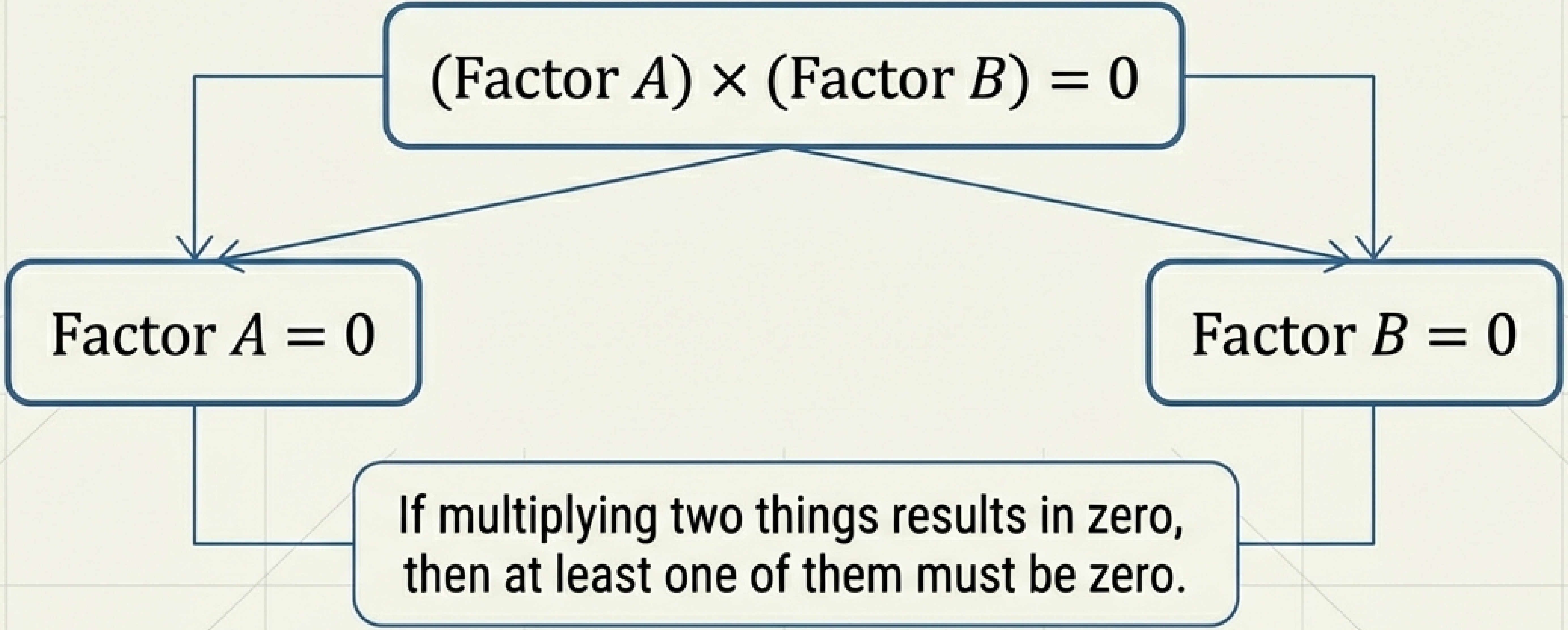
Systematically studied different types of quadratic equations.

Sridharacharya
(India, 1025 C.E.)

Derived the definitive quadratic formula by completing the square.

Tool #1: Solution by Factorisation

A real number (α) is a 'root' if plugging it in makes the equation equal zero: $a(\alpha)^2 + b(\alpha) + c = 0$.



The Factorisation Method in Action

We factorise by splitting the middle term. Let's find
Let's find the roots of $2x^2 - 5x + 3 = 0$.

Step 1: $2x^2 - 5x + 3 = 0$

Step 2: $2x^2 - 2x - 3x + 3 = 0$ → $(-2x) + (-3x) = -5x$, and $(-2x) \times (-3x) = 6x^2$, matching the product of $2x^2$ and 3.

Step 3: Group terms $2x(x - 1) - 3(x - 1) = 0$

Step 4: Extract common factor $(2x - 3)(x - 1) = 0$

Step 5: Equate to zero The roots are $x = \frac{3}{2}$ and $x = 1$

Solving the Prayer Hall Dimensions

Returning to our blueprint equation: $2x^2 + x - 300 = 0$.

$$2x^2 - 24x + 25x - 300 = 0$$

$$(x - 12)(2x + 25) = 0$$

FINAL DIMENSIONS:

Breadth $(x) = 12$ meters

Length $(2x + 1) = 25$ meters

Two possible roots: $x = 12$ OR ~~$x = 12.5$~~ Physical space cannot be negative.

Tool #2: The Quadratic Formula

When an equation is too complex to split the middle term, we use the master key derived by Sridharacharya. It works for every quadratic equation.

$$ax^2 + bx + c = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The Predictor: The Discriminant

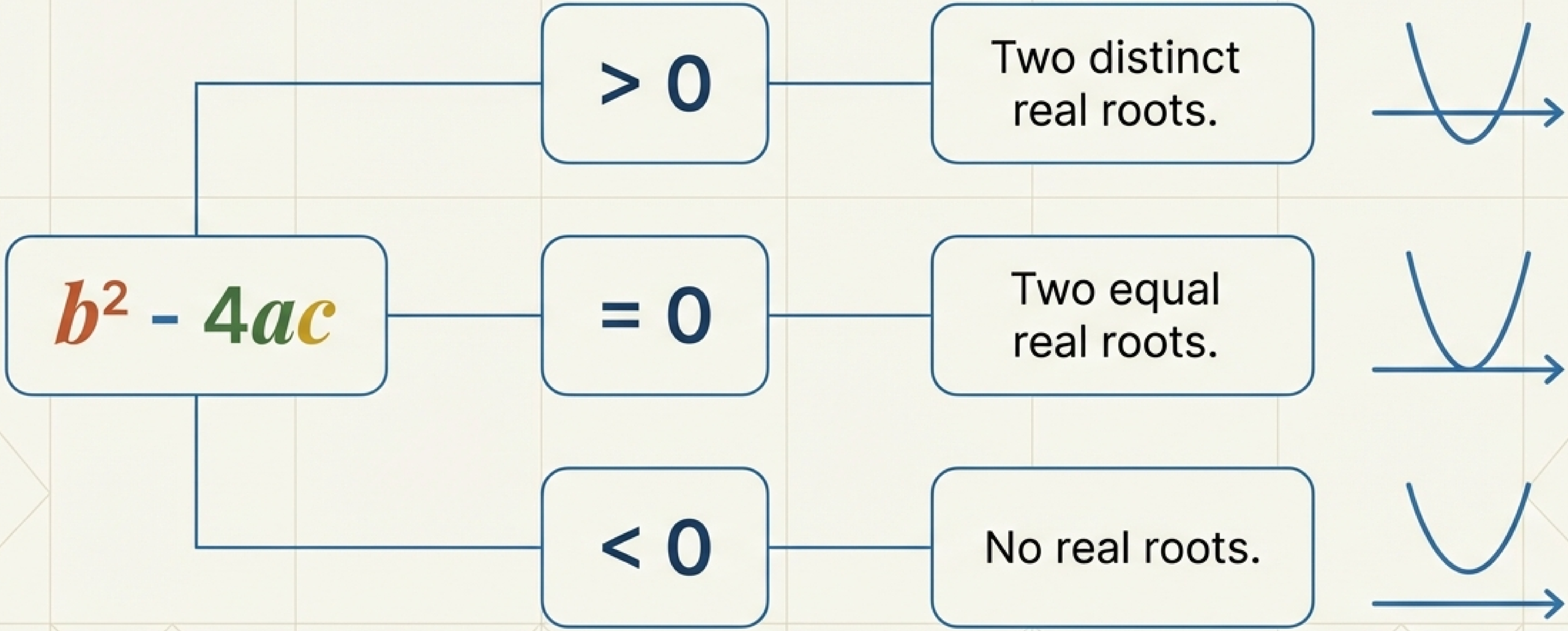
Look closely at the piece of the formula trapped inside the square root.
This is called the Discriminant.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Because we cannot take the square root of a negative number (in the realm of real numbers), this single chunk of math acts as a **crystal ball.** It dictates exactly what kind of answers the equation will produce.

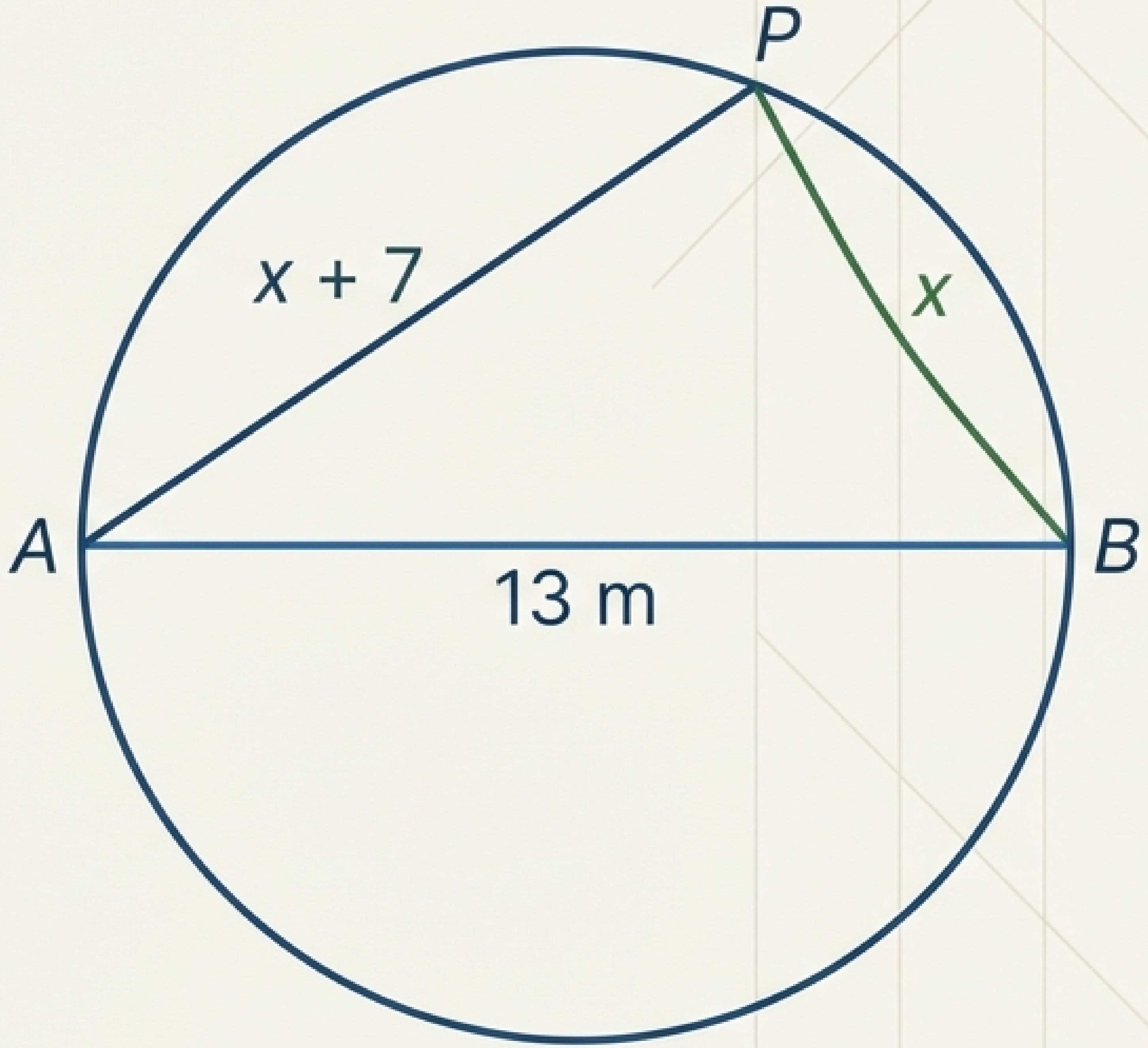
The Nature of Roots

The discriminant ($b^2 - 4ac$) determines the equation's reality.



Application: The Circular Park Problem

A pole must be erected on the boundary of a circular park (diameter 13m). The difference between its distances from two opposite gates (A and B) must be exactly 7m. Is this structurally possible?



Solving the Park Problem

The Equation

Using Pythagoras theorem:

$$(x+7)^2 + x^2 = 13^2$$

Simplifies to:

$$x^2 + 7x - 60 = 0$$

The Predictor

Check the discriminant ($b^2 - 4ac$):

$$7^2 - 4(1)(-60)$$

$$= 49 + 240 = 289$$

Since 289 > 0, real roots exist. It IS possible!

The Solution

Apply the quadratic formula:

$$x = \frac{-7 \pm \sqrt{289}}{2}$$

$$x = \frac{-7 \pm 17}{2}$$

Positive root is x = 5

The pole is 5m from Gate B and 12m from Gate A.

The Quadratic Cheat Sheet

Standard Form

$$ax^2 + bx + c = 0$$

Where a, b, c are real numbers and $a \neq 0$.

Factorisation

Split middle term to create:

$$(x - \alpha)(x - \beta) = 0$$

Roots are $x = \alpha, x = \beta$

Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Works for every quadratic equation.

Discriminant (b^2 - $4ac$)

$$b^2 - 4ac$$

- > 0 : Two distinct real roots
- = 0 : Two equal real roots
- < 0 : No real roots