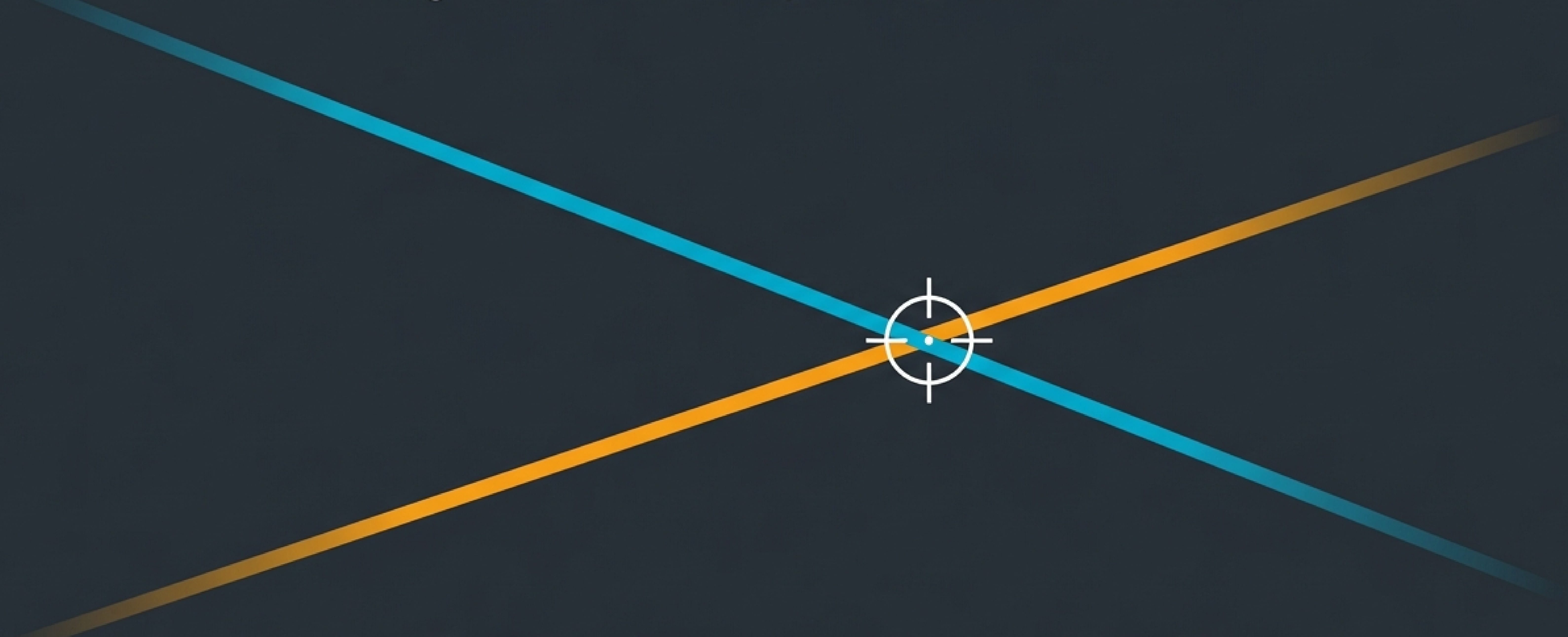
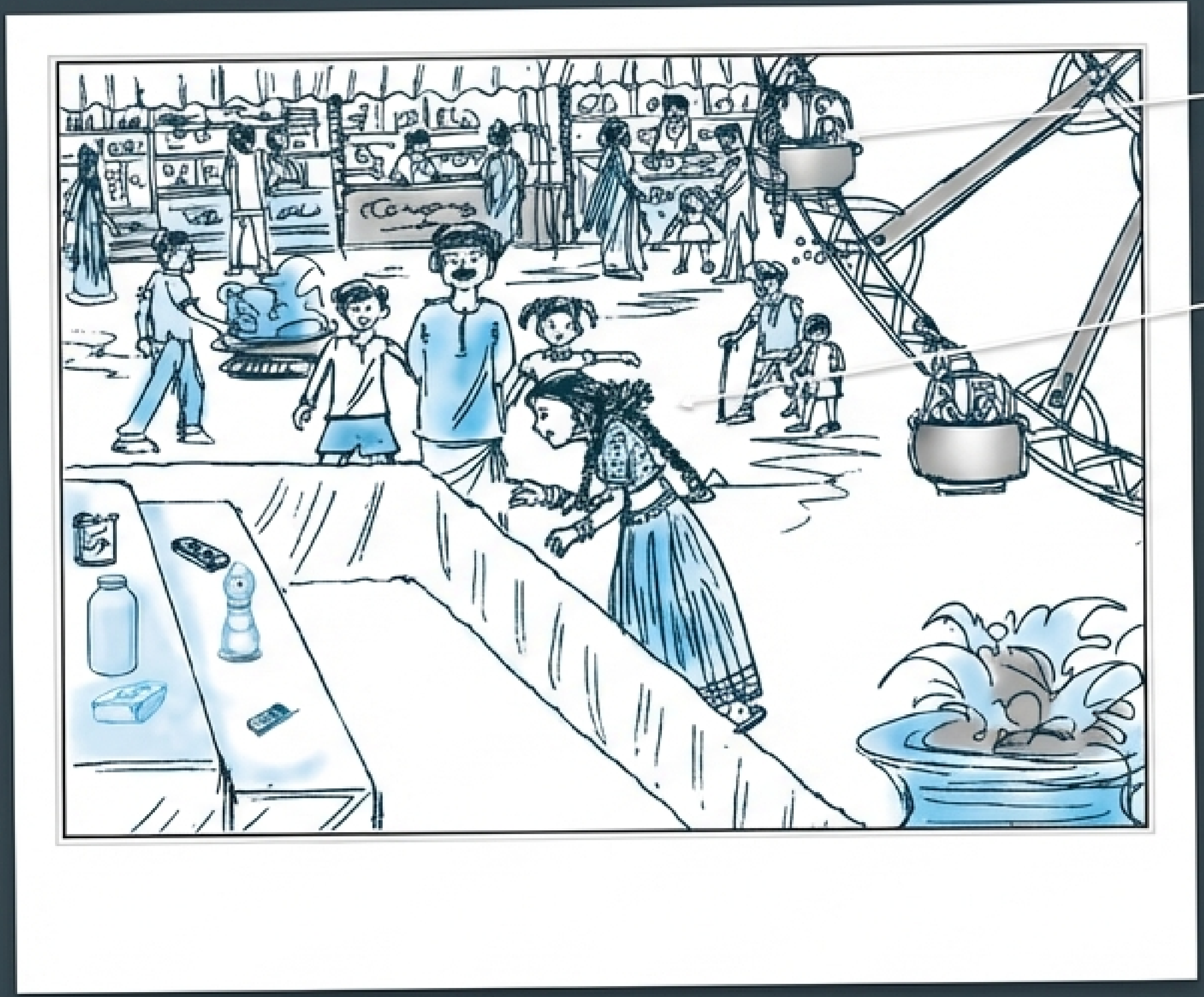
THE TOOLKIT FOR MISSING NUMBERS

Solving Pairs of Linear Equations in Two Variables



In a world of multiple unknowns, a single equation is rarely enough.
To find the truth, we need a pair.

THE MYSTERY AT THE VILLAGE FAIR



Variable x: Giant Wheel Rides (₹3)

Variable y: Hoopla Games (₹4)

Clue 1: Hoopla is half of Rides

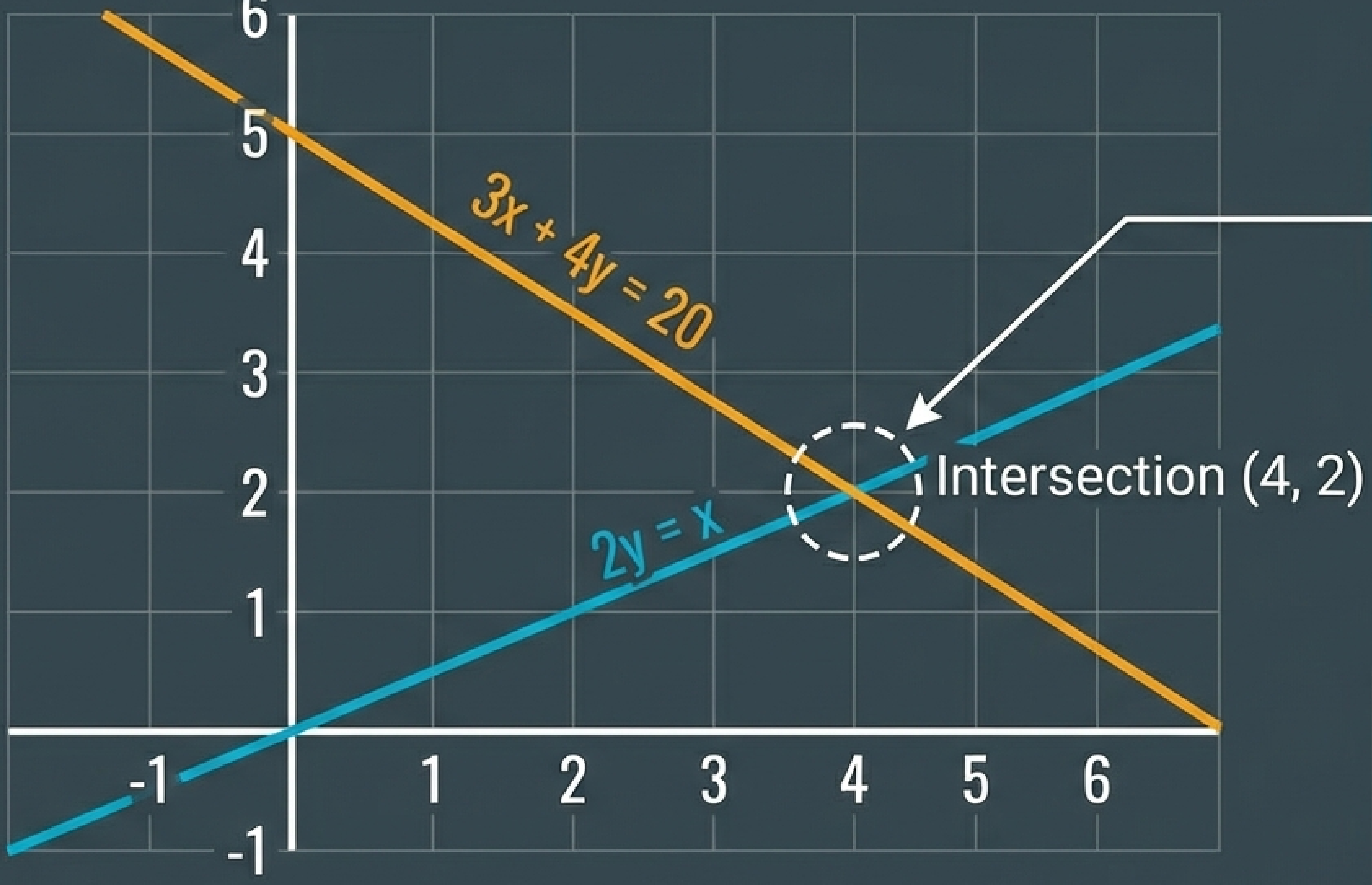
$$y = 1/2\ x$$

Clue 2: Total cost was ₹20

$$3x + 4y = 20$$

The Goal: Find the value for x and y that makes BOTH statements true.

VISUALIZATION: THE GRAPHICAL METHOD

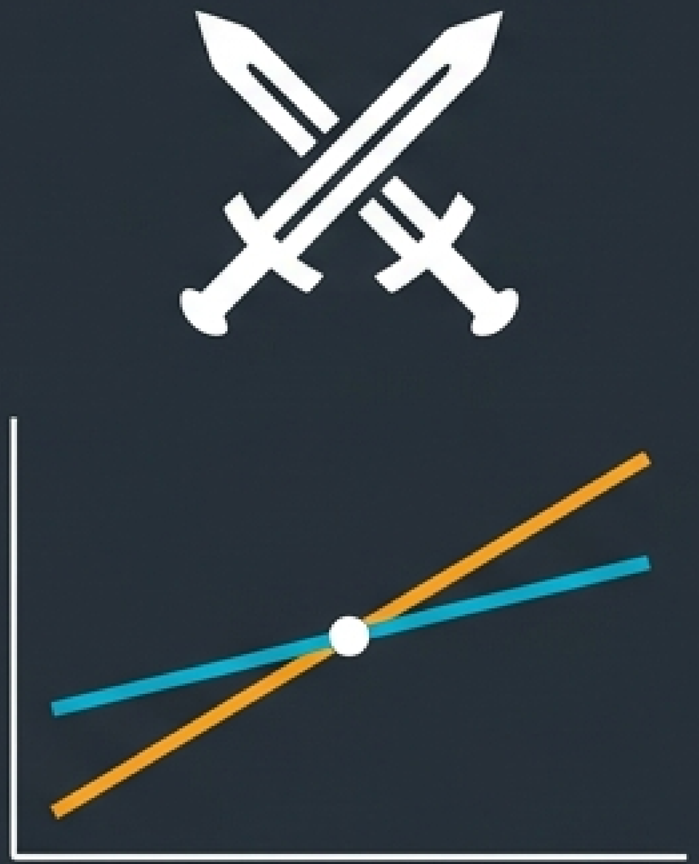


SOLUTION: 4 Rides, 2 Hoopla Games

CONSISTENT: A pair of linear equations that has a solution.

THE THREE BEHAVIORS OF LINES

INTERSECTING



Exactly One Solution.
(Consistent)

PARALLEL

No Solution.
(Inconsistent)

COINCIDENT

Infinitely Many Solutions.
(Dependent/Consistent)

FORENSIC ANALYSIS: THE COEFFICIENT RATIO

For equations $a1x + b1y + c1 = 0$ and $a2x + b2y + c2 = 0$

$$\frac{a1}{a2} \neq \frac{b1}{b2}$$ → Intersecting Lines (Unique Solution)

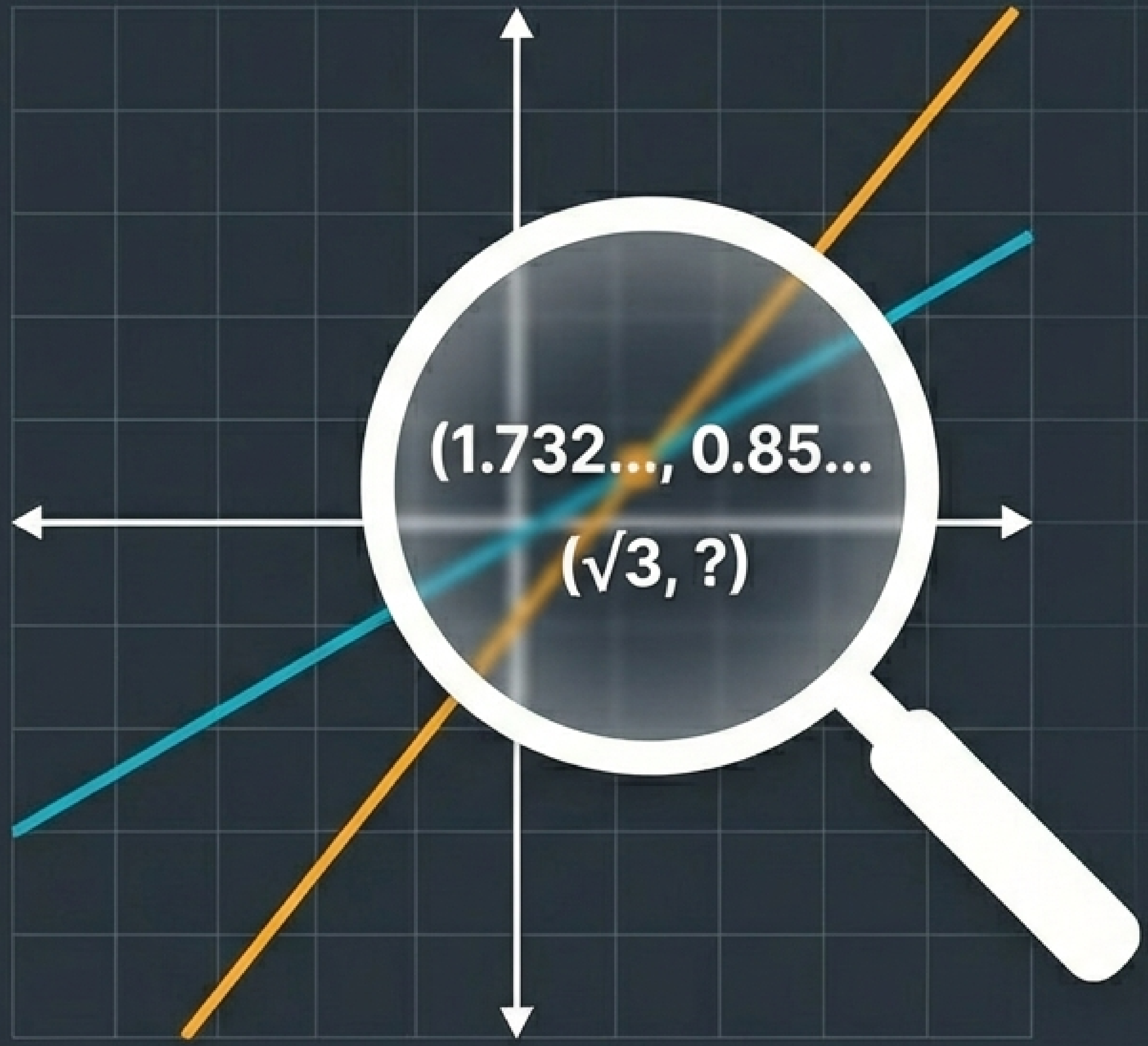
$$\frac{a1}{a2} = \frac{b1}{b2} \neq \frac{c1}{c2}$$ → Parallel Lines (No Solution)

$$\frac{a1}{a2} = \frac{b1}{b2} = \frac{c1}{c2}$$ → Coincident Lines (Infinite Solutions)

CASE STUDY: PREDICTING THE OUTCOME

Equation Pair	The Check	Verdict
$x - 2y = 0$ $3x + 4y - 20 = 0$	$1/3 \neq -2/4$	INTERSECTING
$2x + 3y - 9 = 0$ $4x + 6y - 18 = 0$	$2/4 = 3/6 = -9/-18$ (All equal 1/2)	COINCIDENT
$x + 2y - 4 = 0$ $2x + 4y - 12 = 0$	$1/2 = 2/4 \neq -4/-12$ (Equals 1/2) (Does not equal 1/3)	PARALLEL

WHY WE NEED ALGEBRA



The Problem: Graphs are excellent for behavior, but **dangerous** for precision.

The Reality: Coordinates like (√3, 1.75) are **impossible** to read accurately by eye.

The Solution: To find exact values, we need **Algebraic Methods**.

TOOL #1: THE SUBSTITUTION METHOD

Strategy: Isolate & Replace

The Algorithm

1. Pick one equation.
2. Isolate one variable (e.g., $x = \ldots$).
3. **SUBSTITUTE** this value into the other equation.
4. Solve and back-substitute.

The Live Example

Equation 1: $7x - 15y = 2$

Equation 2: $x + 2y = 3$

→ $x = \mathbf{3 - 2y}$

Equation 1:

$$7(\mathbf{3 - 2y}) - 15y = 2$$

SUBSTITUTION IN ACTION: AFTAB'S AGE

STEP 1: SETUP

$s - 7 = 7(t - 7)$ (Eq 1)

$s + 3 = 3(t + 3)$ (Eq 2)

↓

STEP 2: ISOLATE

$s = 3t + 6$

↓

STEP 3: SUBSTITUTE

$(3t + 6) - 7 = 7(t - 7)$

STEP 3: SUBSTITUTE

$(3t + 6) - 7 = 7(t - 7)$

↓

STEP 4: SOLVE

$3t - 1 = 7t - 49$

$-4t = -48$

$t = 12$

↓

STEP 5: FINAL ANSWER

✔ Daughter is 12. Aftab is 42.

TOOL #2: THE ELIMINATION METHOD

Strategy: Level & Remove

The Algorithm

1. Multiply equations to match coefficients.
2. Add or Subtract to ELIMINATE one variable.
3. Solve the remaining simple equation.

The Live Example

Equation 1: $9x - 4y = 2000$

$\times 3$

Equation 2: $7x - 3y = 2000$

$\times 3$ $27x - 12y = 6000$

$\times 4$ $28x - 12y = 8000$

$x = 2000$

ELIMINATION IN ACTION: REVERSED DIGITS

Sometimes coefficients align naturally.

$$
\begin{array}{r}
x + \cancel{y} = 6 \\
+\ x - \cancel{y} = 2 \\
\hline
2x = 8 \longrightarrow x = 4
\end{array}
$$

If $x = 4$, then $y = 2$. The number is 42.

WHEN VARIABLES VANISH

What happens when the math breaks?

The False Statement

$$-4 = 0$$

Parallel Lines
(No Solution)

The True Statement

$$18 = 18$$

Coincident Lines
(Infinite Solutions)

THE LINEAR EQUATIONS SUMMARY

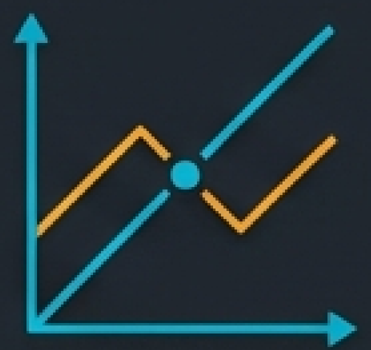
Graph Visual	Algebraic Ratio	Solutions	Consistency
	$\frac{a1}{a2} \neq \frac{b1}{b2}$	Unique	Consistent
	$\frac{a1}{a2} = \frac{b1}{b2} = \frac{c1}{c2}$	Infinite	Dependent (Consistent)
	$\frac{a1}{a2} = \frac{b1}{b2} \neq \frac{c1}{c2}$	None	Inconsistent

CHOOSING YOUR WEAPON

When to use which method?

GRAPHICAL

Use for visualizing trends or checking integer answers.



SUBSTITUTION

Use when one variable is already isolated (e.g., $y = 4x + 1$).

$$y = 4x + 1$$

ELIMINATION

Use when coefficients are easy to balance (e.g., $3x$ and $-3x$).

$$\begin{array}{r} 3x \\ -3x \\ \hline 0 \end{array}$$

NAVIGATING THE UNKNOWN

Whether the lines cross, overlap, or run parallel, we now have the tools to define their relationship. Linear equations allow us to navigate a world with multiple unknowns.