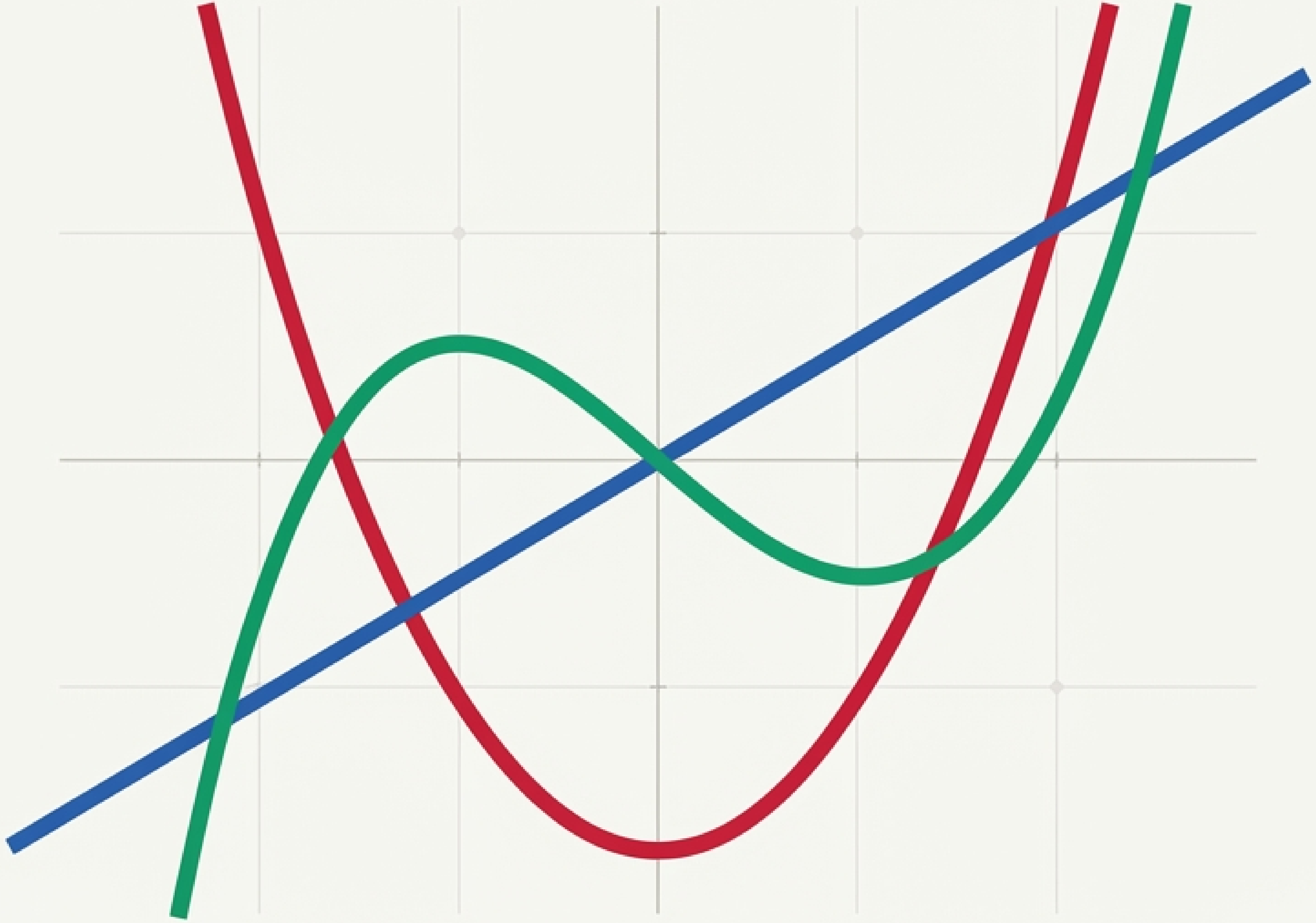
The Anatomy of Polynomials

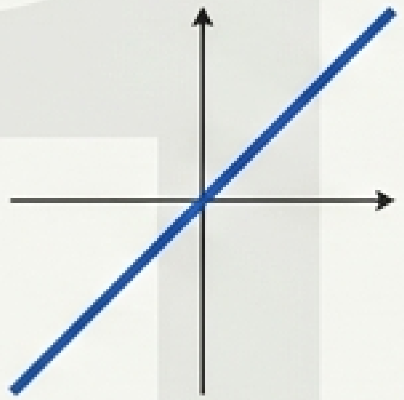
A visual guide to degrees, zeroes, and coefficients.



BASED ON MATHEMATICS CHAPTER 2

Degrees of Complexity

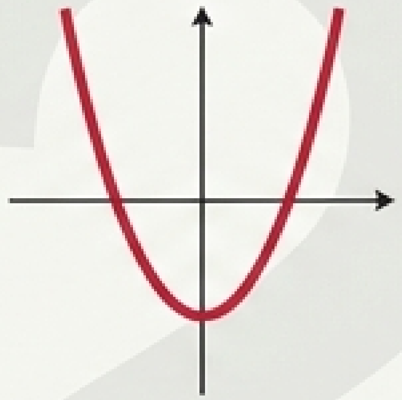
Linear



Degree: 1

$2x - 3$

Quadratic

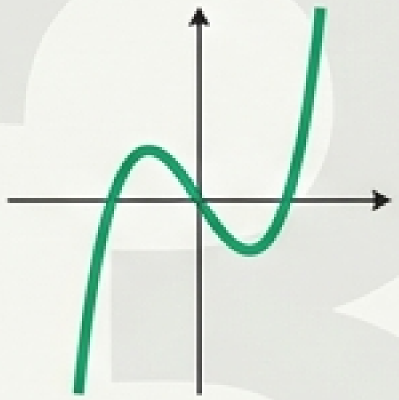


Degree: 2

$x^2 - 3x - 4$

Etymology: From “Quadrate” meaning Square.

Cubic



Degree: 3

$x^3 - 4x$

What is a Zero?

A real number k is a zero of a polynomial $p(x)$ if $p(k) = 0$.

$$p(x) = x^2 - 3x - 4$$

When $x = -1$

$$(-1)^2 - 3(-1) - 4$$

$$1 + 3 - 4 = 0$$

When $x = 4$

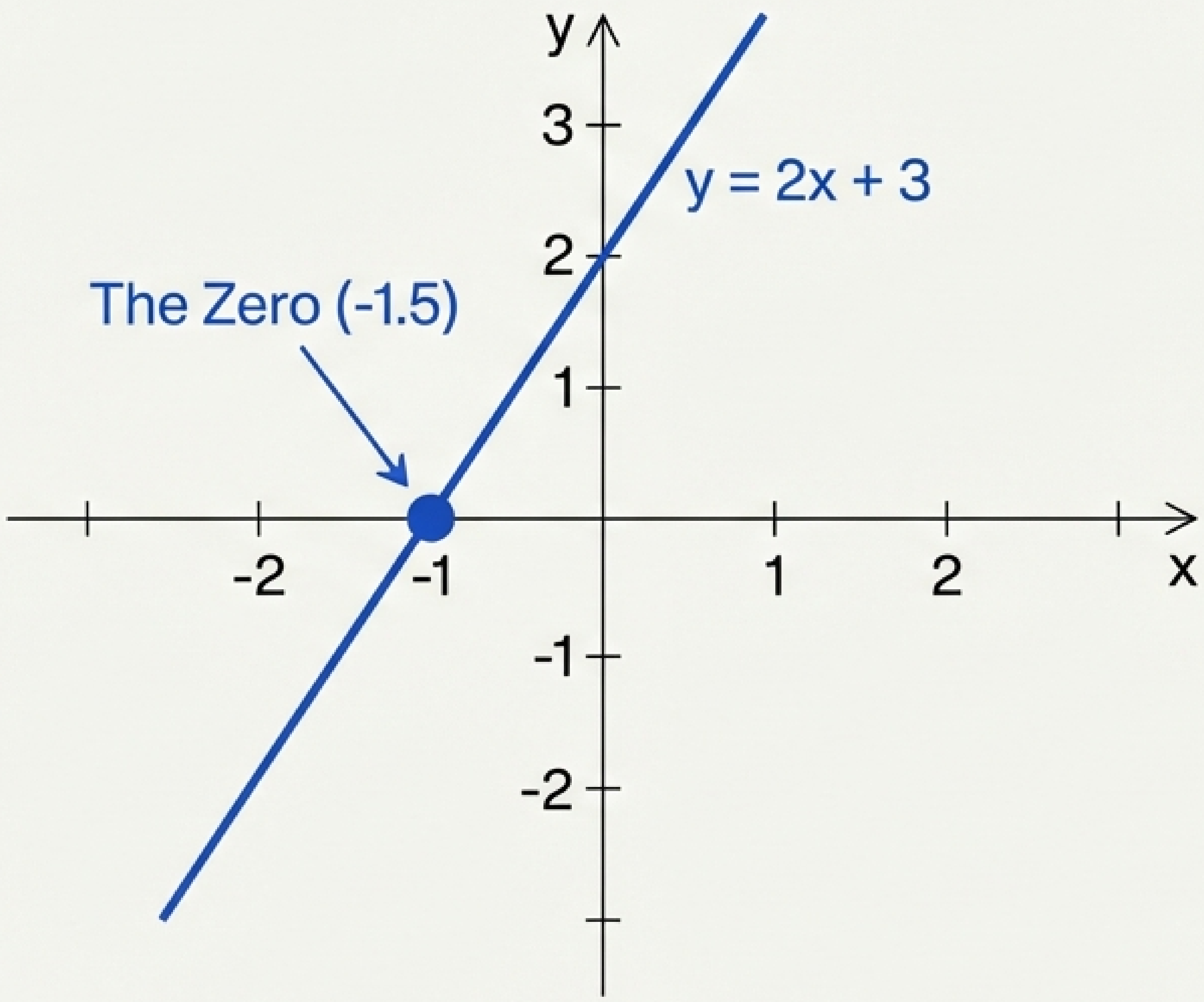
$$4^2 - 3(4) - 4$$

$$16 - 12 - 4 = 0$$

Mathematical Translation: ‘Zero’ essentially means ‘Solution’. Here, -1 and 4 are the zeroes.

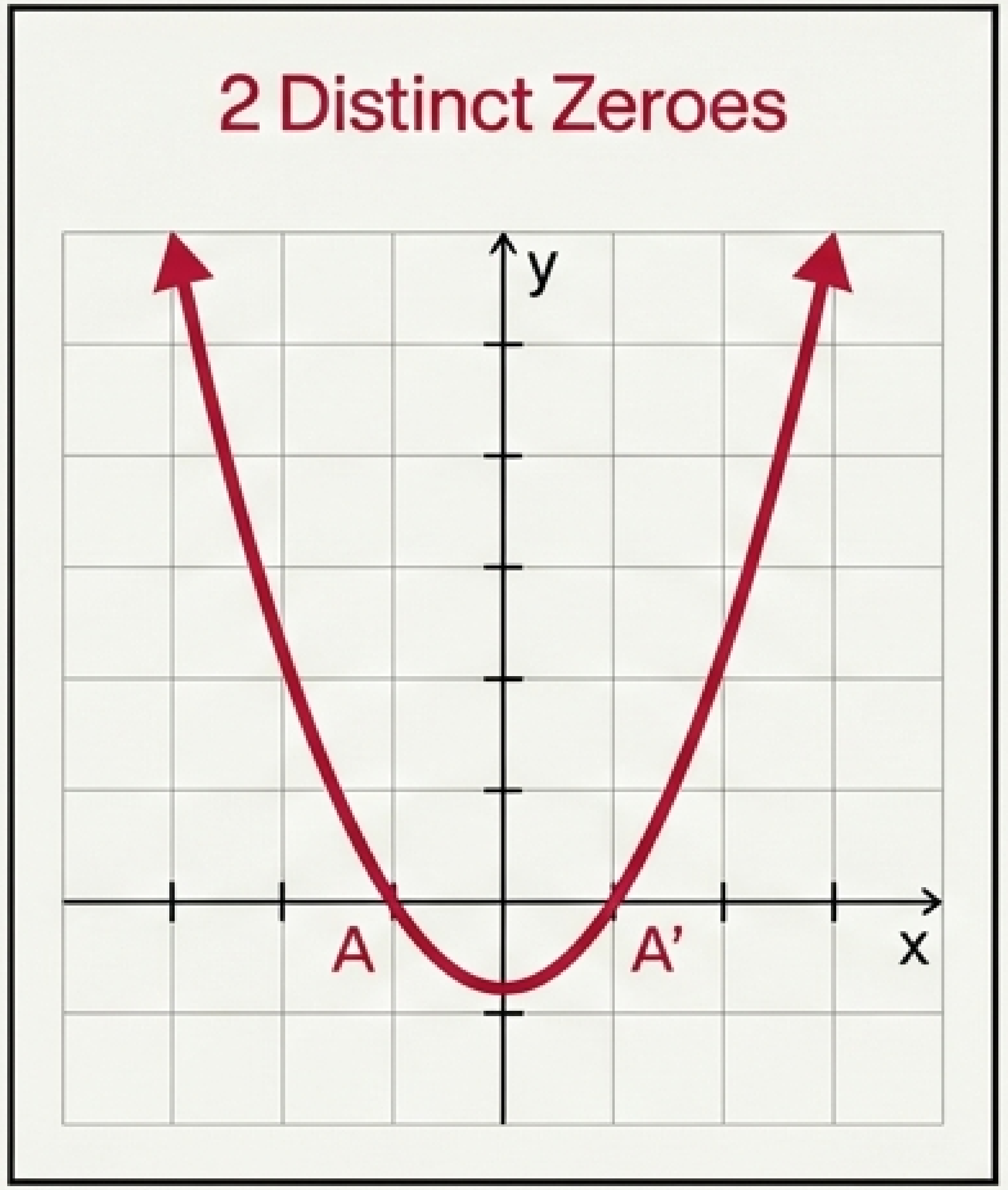
Geometry of the Zero

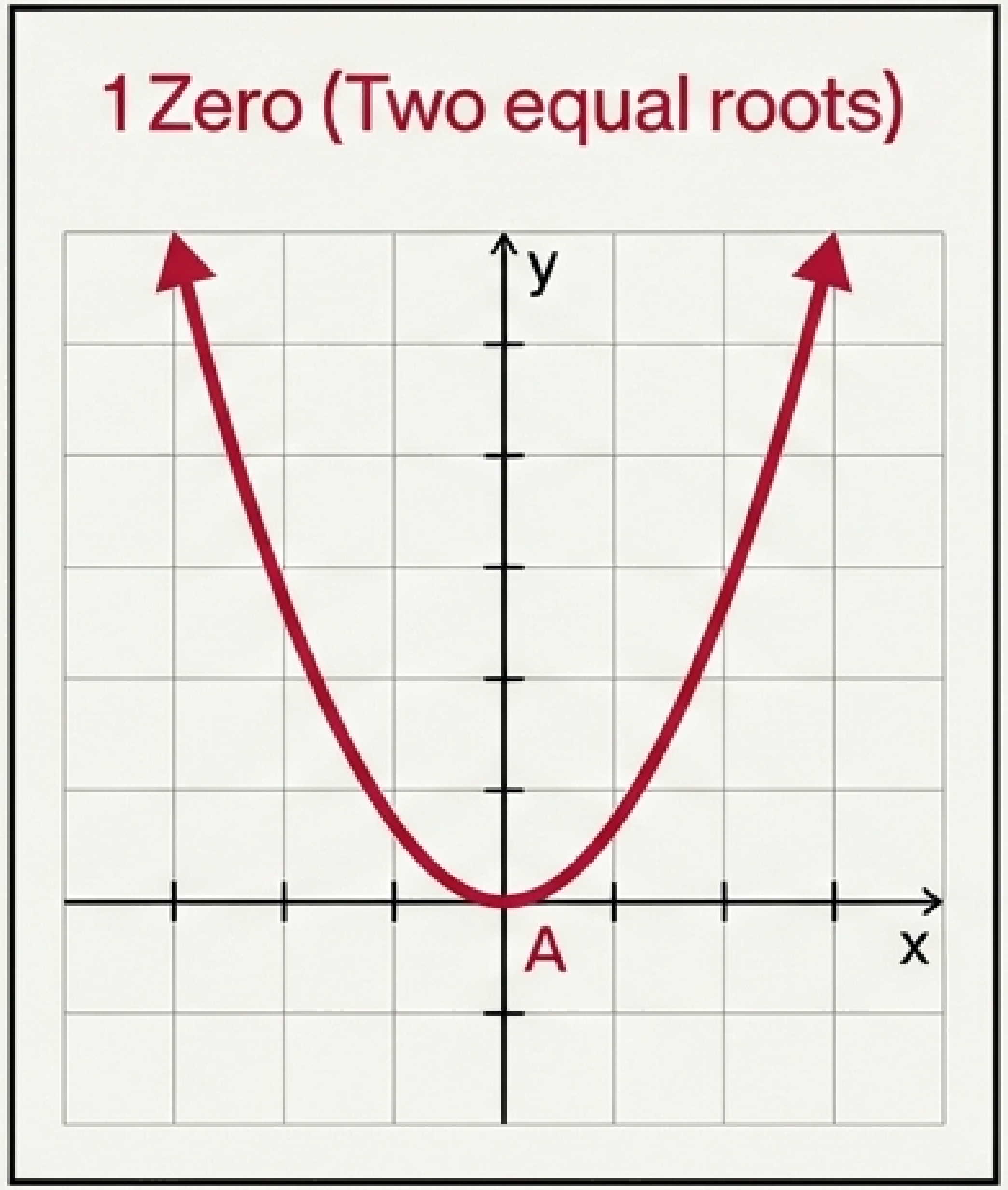
The **zero** of a polynomial is the **x-coordinate** where the graph intersects the **x-axis.**

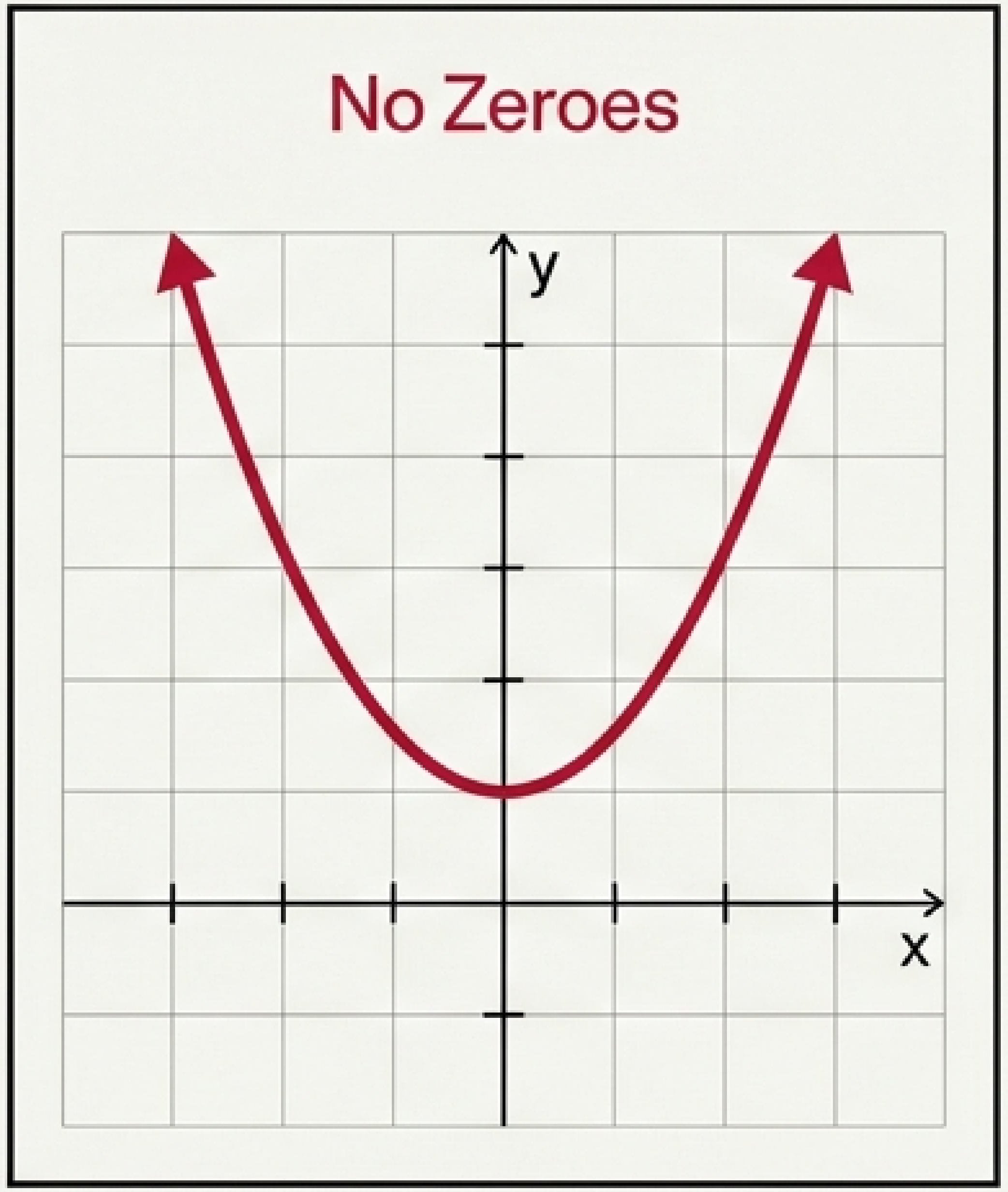


The Shape of Quadratics

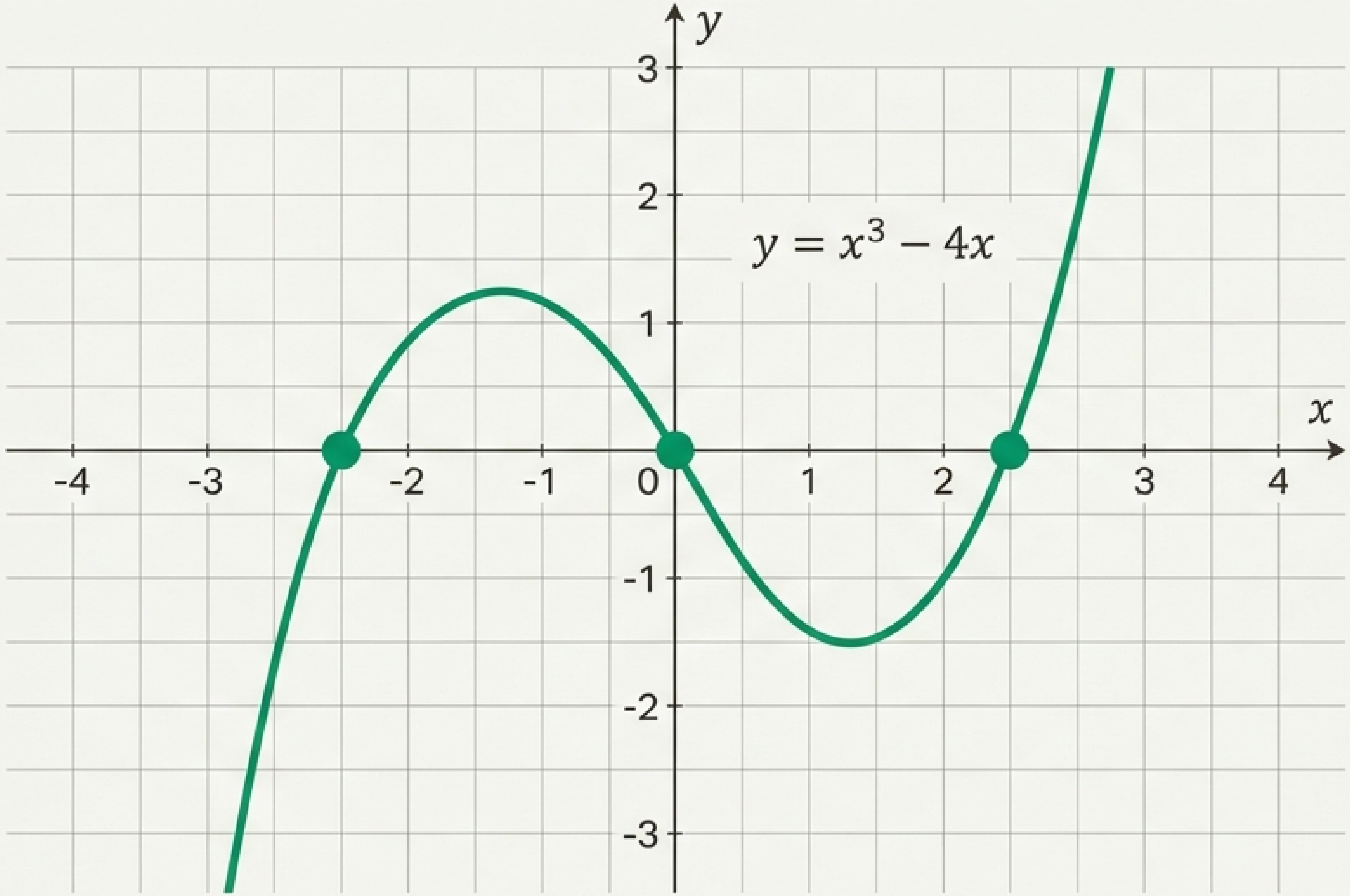
The curve $y = ax^2 + bx + c$ is known as a **Parabola.**







Degrees vs. Zeroes



Golden Rule

A polynomial of degree *n* has **at most** *n* zeroes.

- Degree 1 (Linear) → Max 1 Zero
- Degree 2 (Quadratic) → Max 2 Zeroes
- Degree 3 (Cubic) → Max 3 Zeroes

The Hidden Relationship: Quadratic

$$ax^2 + bx + c$$

Sum of Zeroes (α + β)	Product of Zeroes (αβ)
$= \frac{-b}{a}$	$= \frac{c}{a}$
Negative Coefficient of x ÷ Coefficient of x^2	Constant Term ÷ Coefficient of x^2

Relationship in Action

Polynomial: $p(x) = x^2 + 7x + 10$

Step 1: Find Zeroes

$$x^2 + 7x + 10$$

$$\rightarrow = (x+2)(x+5)$$

Zeroes are **-2** and **-5**

Step 2: Verify Formula

Sum:

$(-2) + (-5) =$ **-7**

Formula: $\frac{-b}{a} = \frac{-7}{1} =$ **-7**

Product:

$(-2) \times (-5) =$ **10**

Formula: $\frac{c}{a} = \frac{10}{1} =$ **10**

The Hidden Relationship: Cubic

For equation $ax^3 + bx^2 + cx + d$ with zeroes α, β, γ

Sum of Zeroes
$\alpha + \beta + \gamma = -\frac{b}{a}$
Sum of Product Pairs
$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}$
Product of Zeroes
$\alpha\beta\gamma = -\frac{d}{a}$

α (Alpha), β (Beta), γ (Gamma) = The three zeroes

Polynomials at a Glance

Geometric Identity

Zeroes are physically the **x-intercepts** of the graph.

The Constraint

The maximum number of zeroes equals the **Degree** of the polynomial.

Quadratic Decoder

Sum of zeroes = **$-b/a$**

Product of zeroes = **c/a**

The algebra predicts the geometry. The coefficients control the zeroes.