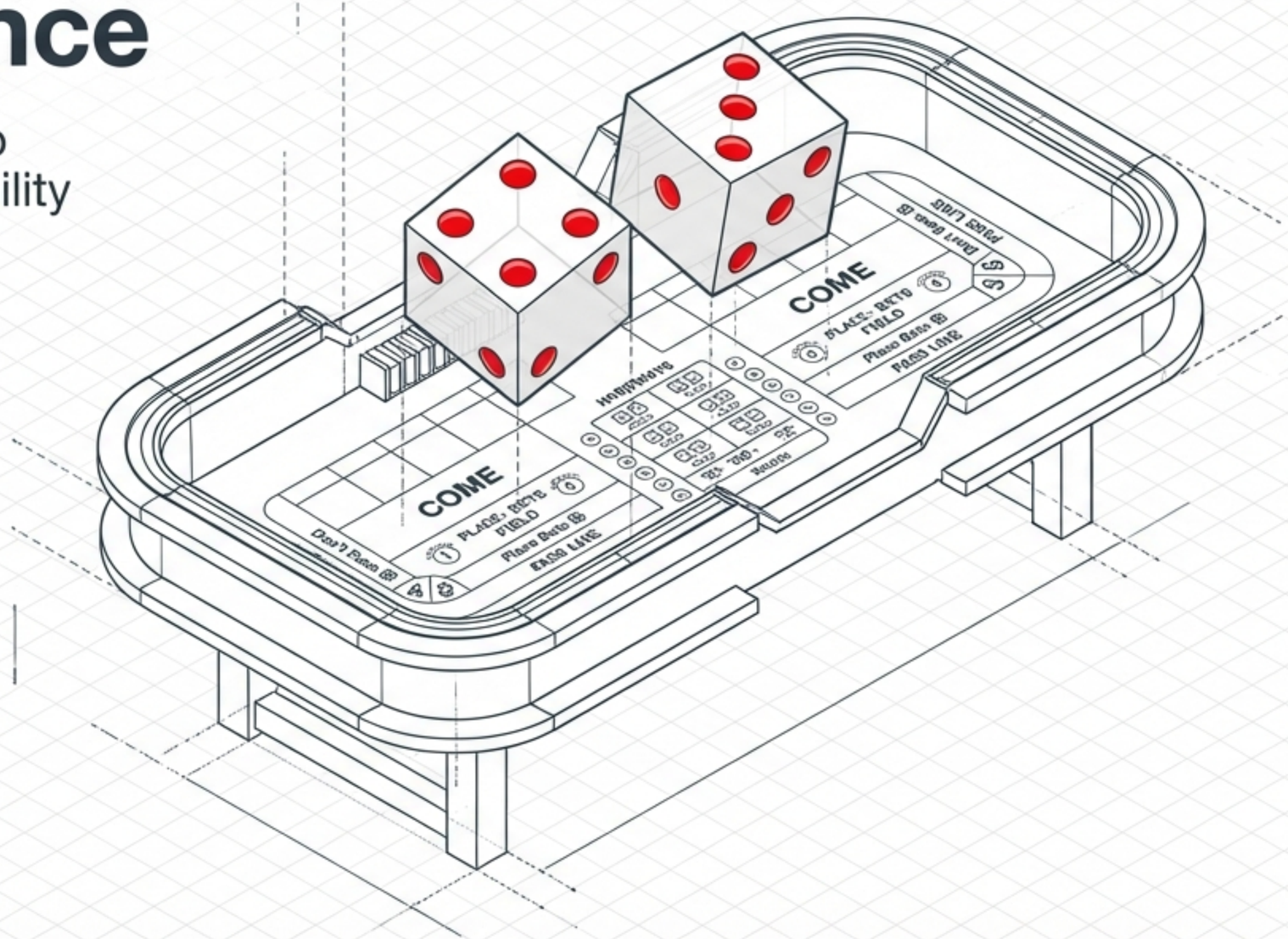
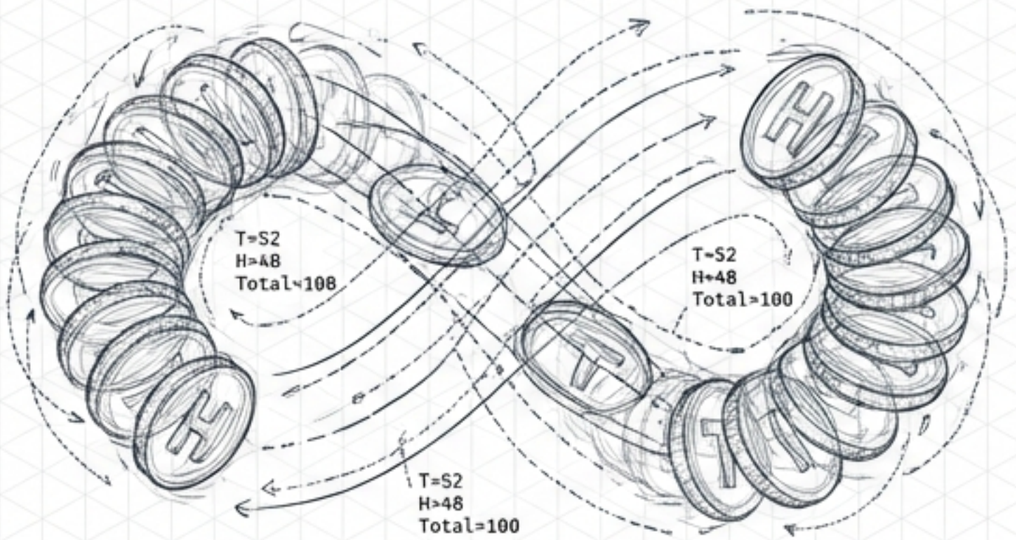
The Rules of Chance

A Visual Guide to Classical Probability



Predicting the Future Without Repeating the Past

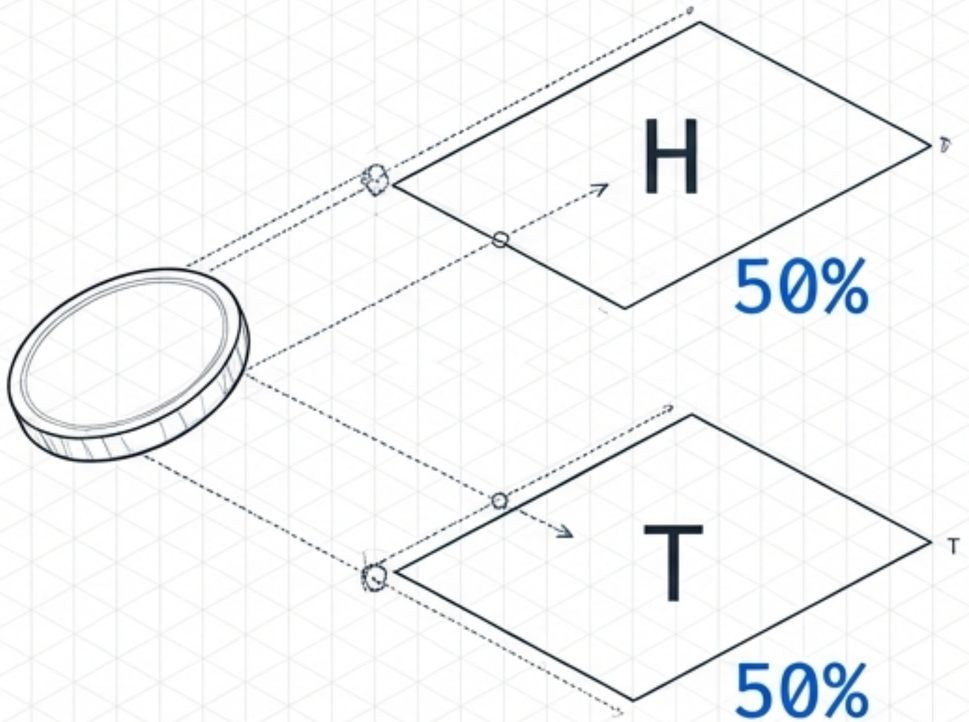
Empirical (Experimental)



$$P(E) = \frac{\text{Trials where event happened}}{\text{Total number of trials}}$$

⚠ Unfeasible for high-stakes: You cannot crash-test a satellite repeatedly to find its failure rate.

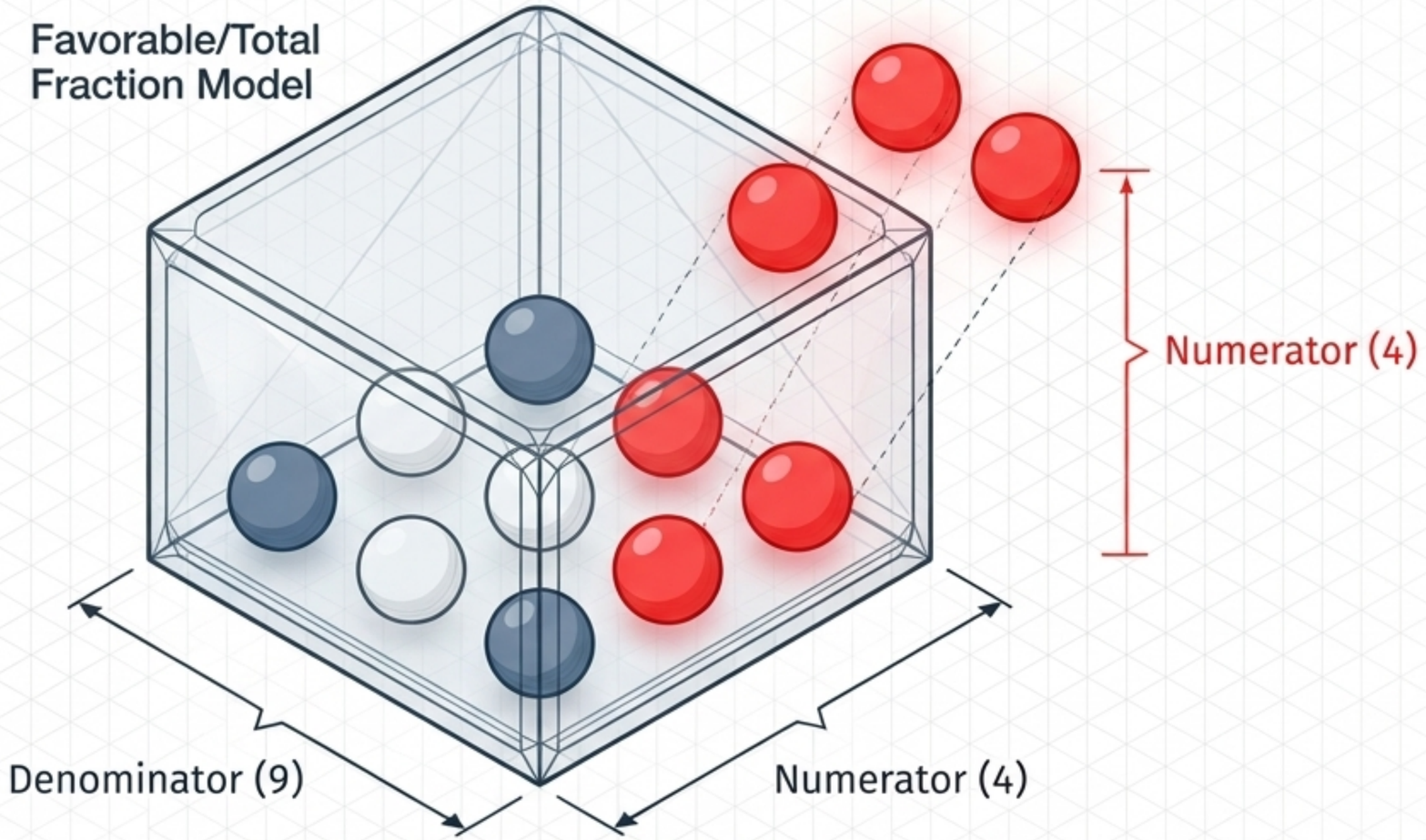
Theoretical (Classical)



Pierre Simon Laplace (1795). Predicts exact likelihood based on equally likely assumptions. No repetition required.

The Master Equation

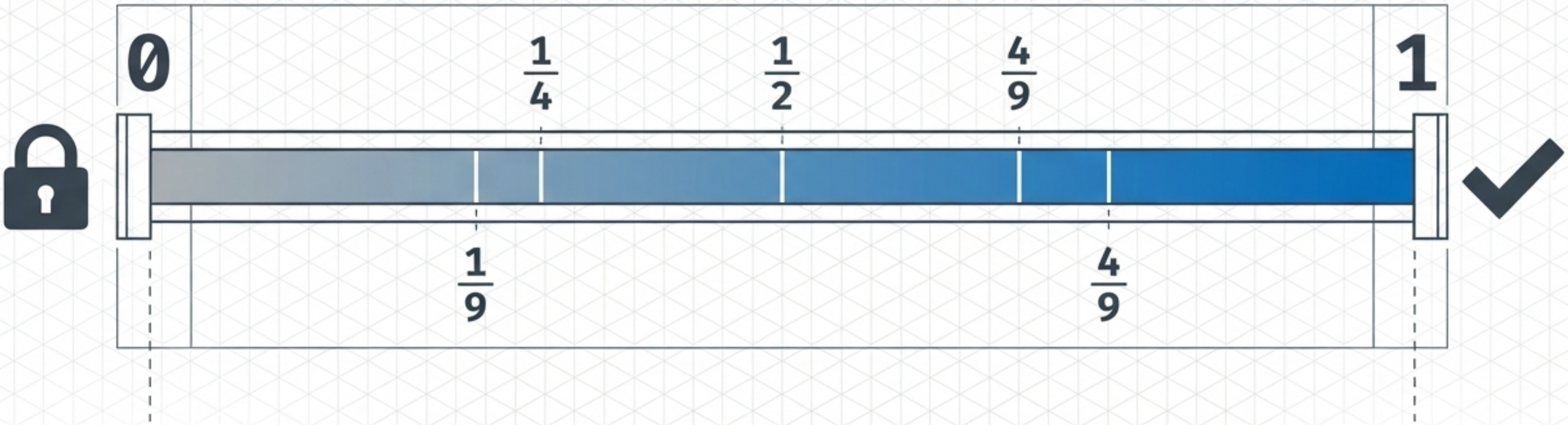
$$P(E) = \frac{\text{Number of outcomes favourable to } E}{\text{Number of all possible outcomes}}$$



Example: P(Red Marble) = 4/9

The Probability Spectrum

$$0 \leq P(E) \leq 1$$



Impossible Event

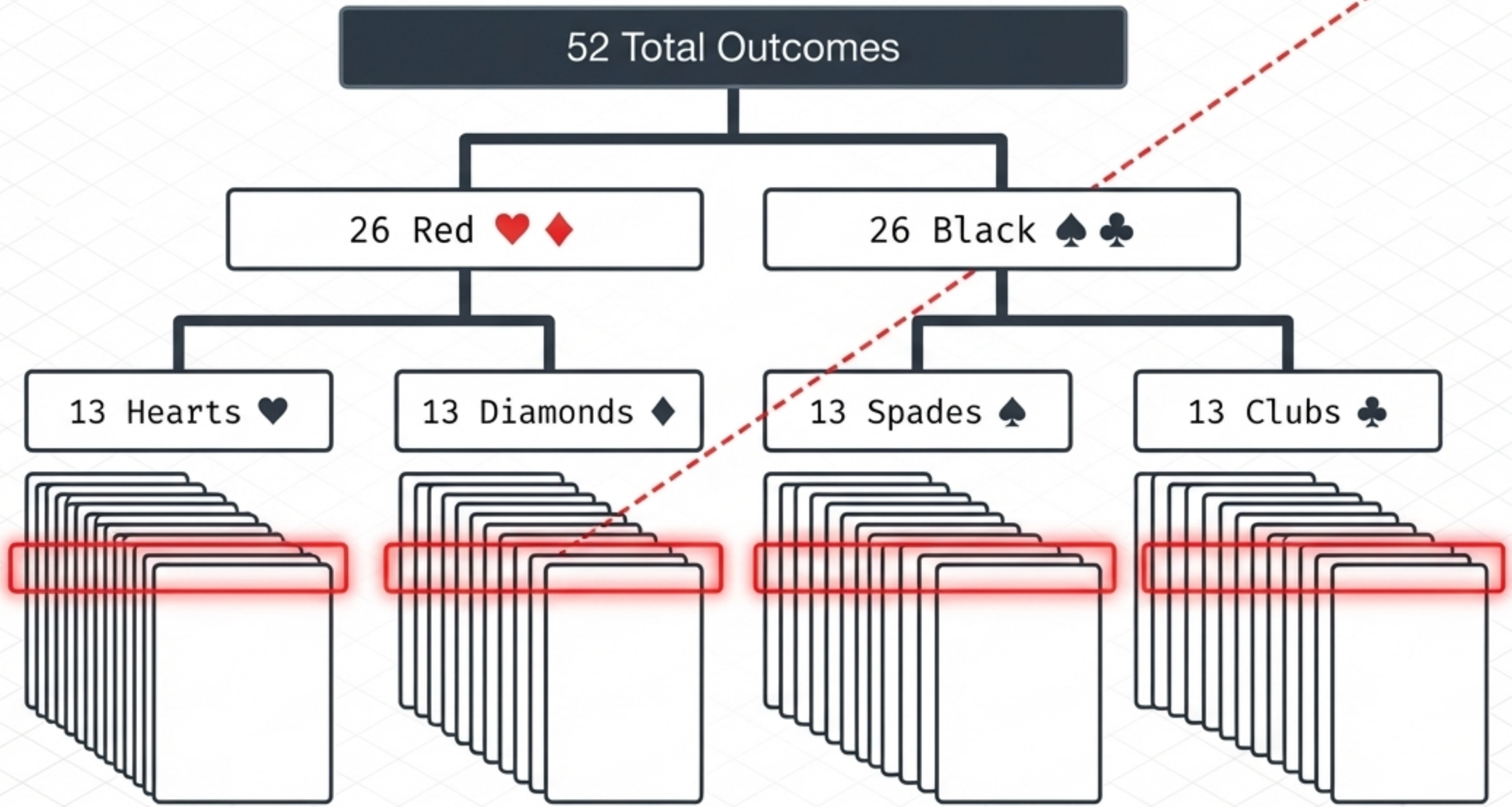
Rolling an 8 on a standard 6-sided die.

Sure Event

Rolling a number less than 7.

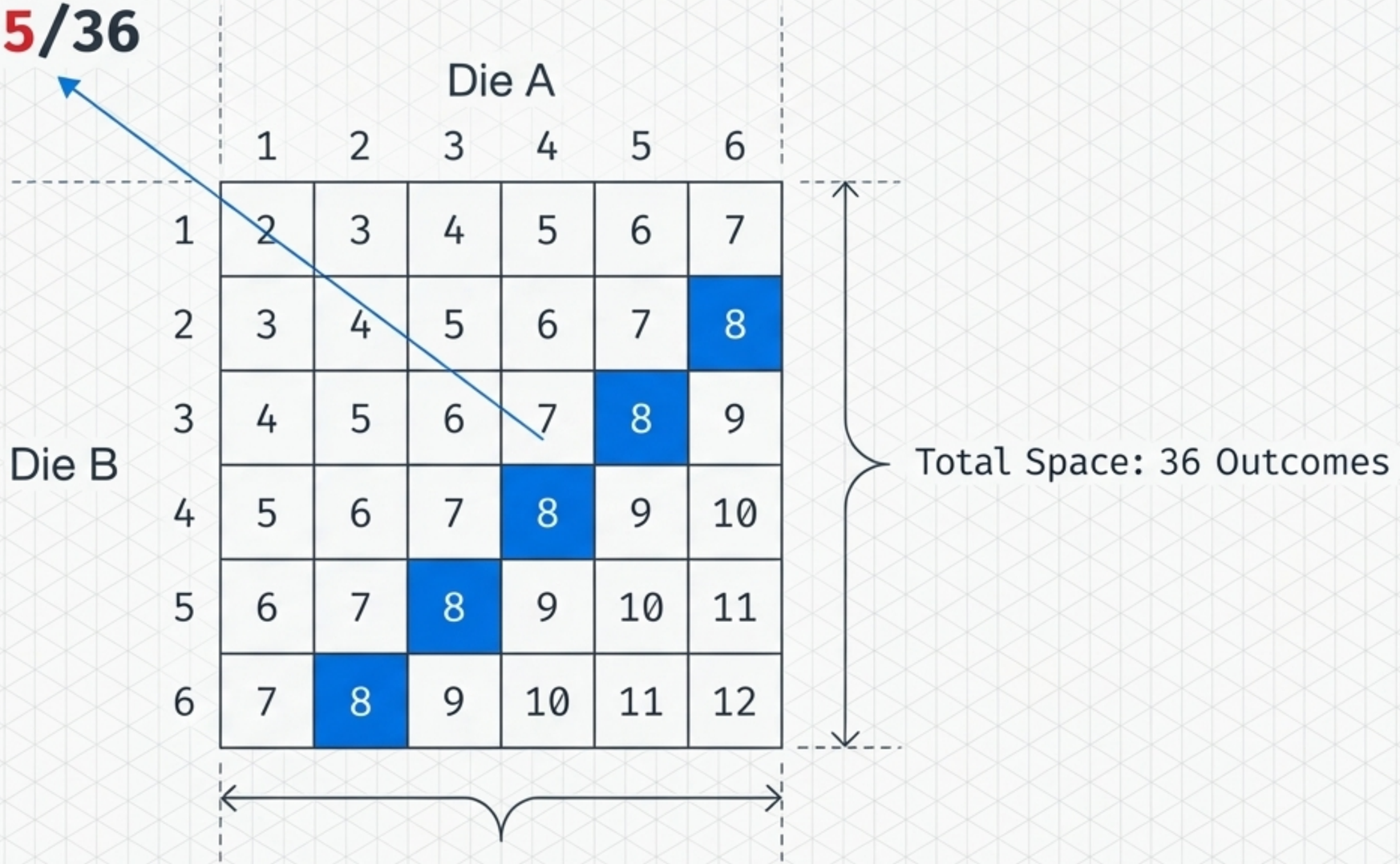
System Anatomy: The 52-Card Deck

$$P(\text{Drawing an Ace}) = \frac{4}{52} = \frac{1}{13}$$



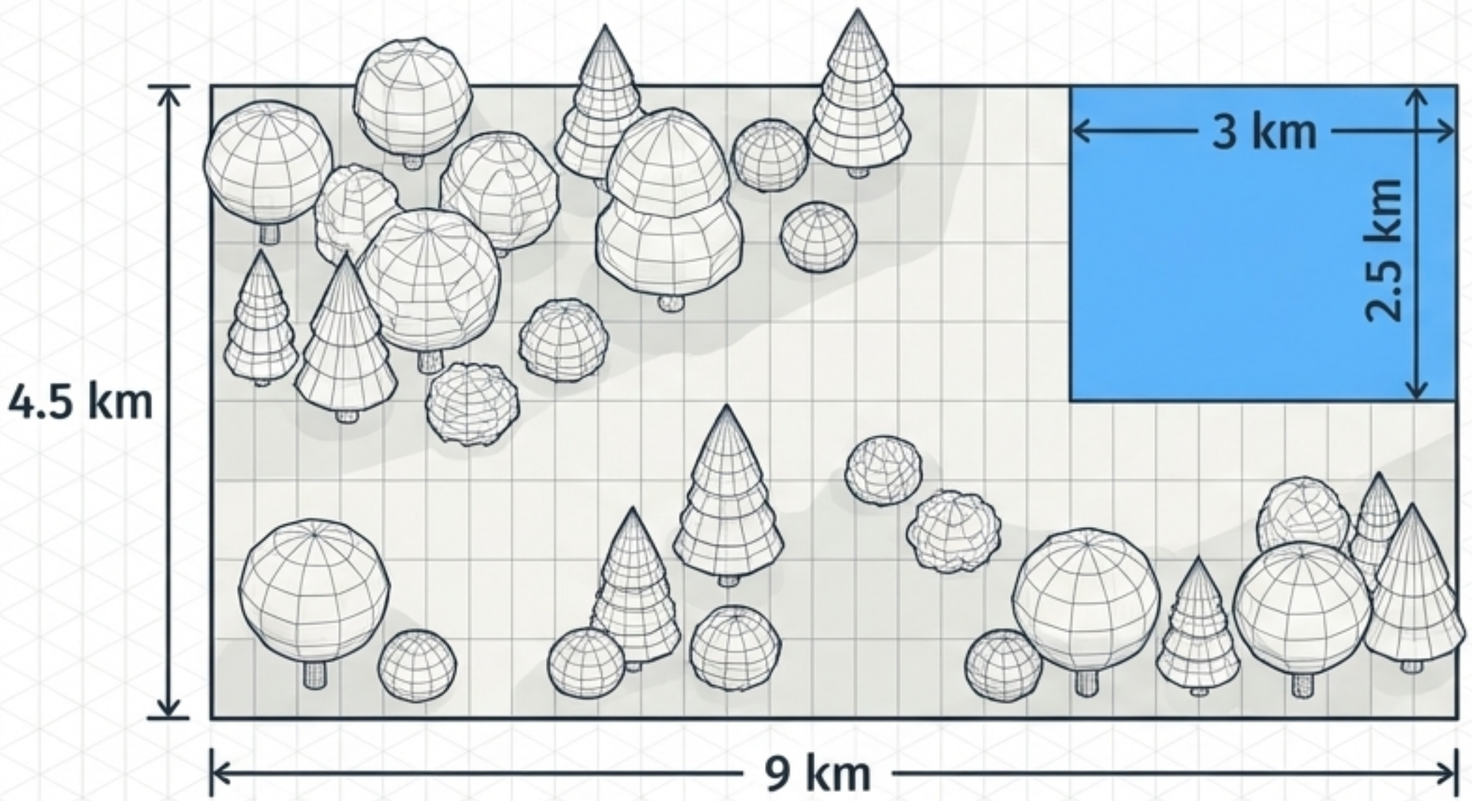
The Matrix of Two Dice

P(Sum of 8) = 5/36



Geometric Probability

A helicopter crashes within a 40.5 km^2 region.
What is the probability it landed in the lake?



Total Area = 40.5 km^2

$$P(\text{Lake}) = \frac{\text{Area of Lake}}{\text{Total Area}}$$

$$P(\text{Lake}) = \frac{7.5}{40.5}$$

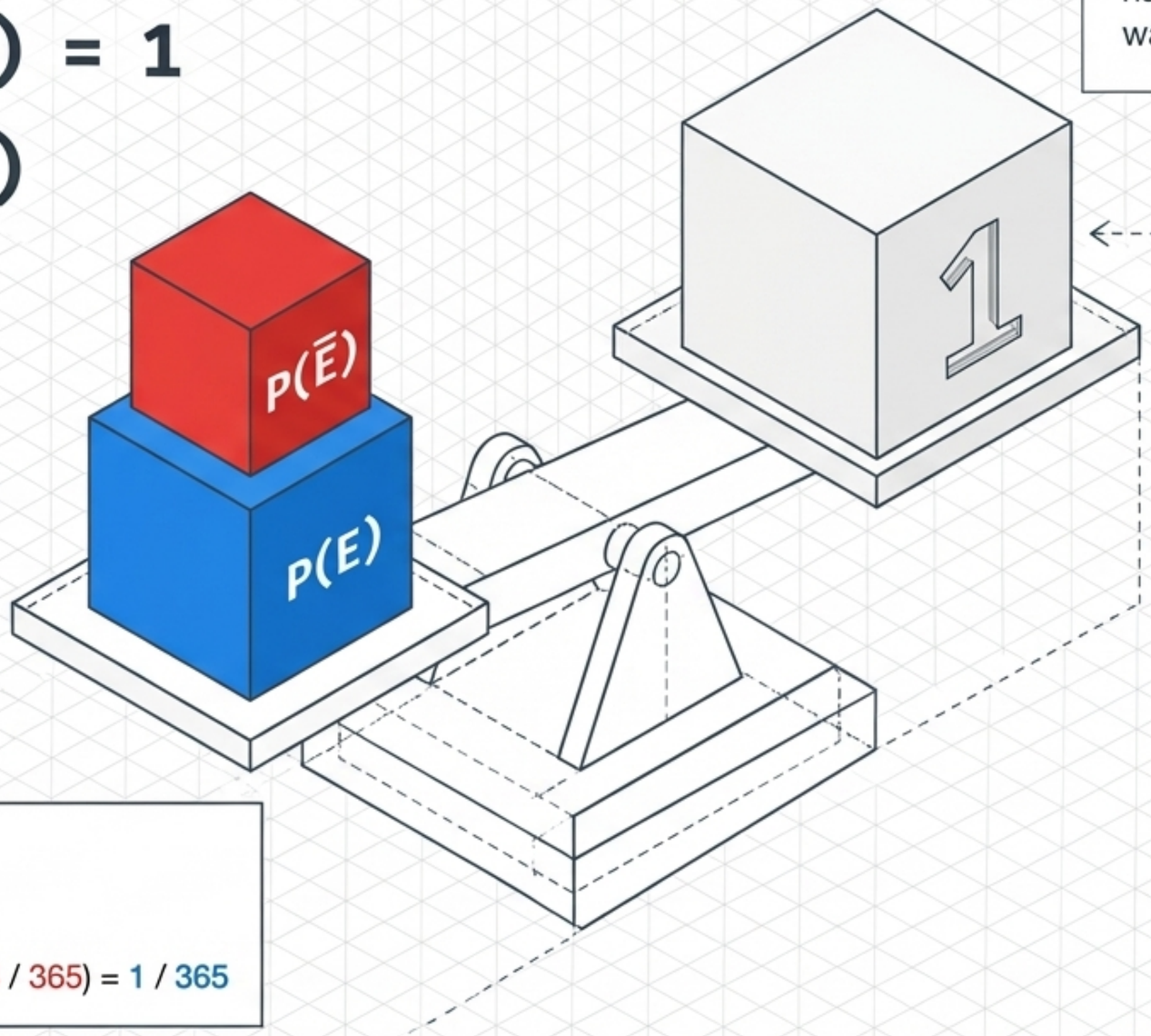
$$P(\text{Lake}) = \frac{5}{27}$$

The Law of Complements

$$P(E) + P(\text{not } E) = 1$$

$$P(\bar{E}) = 1 - P(E)$$

Solving for what *won't* happen is often the fastest way to solve for what *will*.



Example: Savita and Hamida

P(Different Birthdays) = 364 / 365
Therefore, P(Same Birthday) = 1 - (364 / 365) = 1 / 365