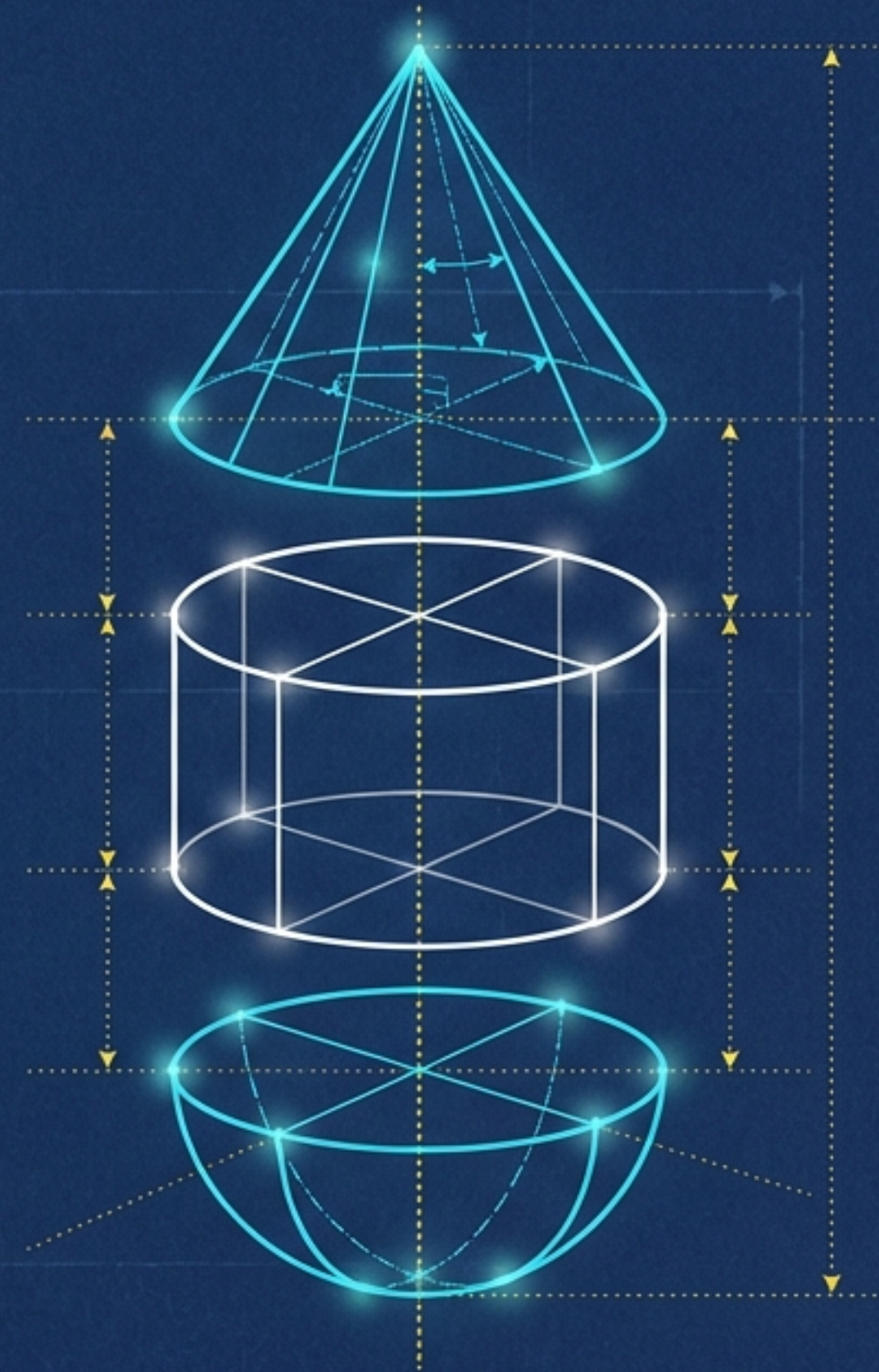


Building Blocks: Mastering Surface Area & Volume of Combined Solids

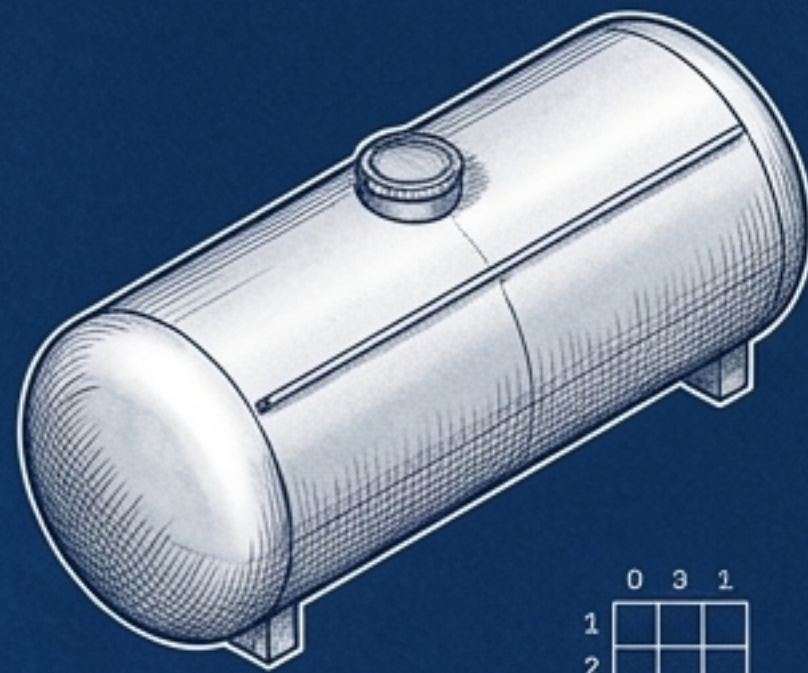
A visual guide to deconstructing
complex 3D shapes.



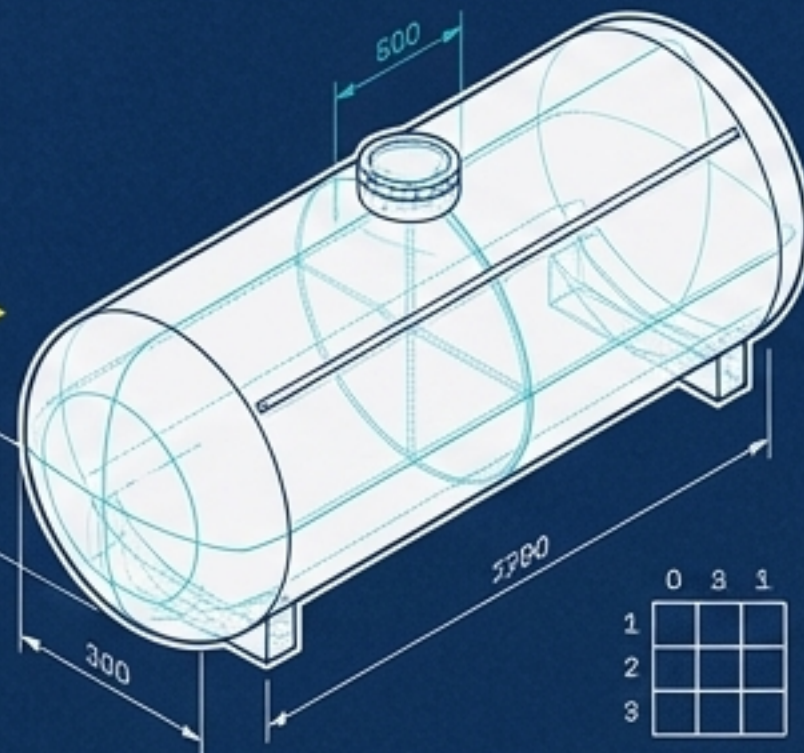
The Core Principle: Deconstruction

Every complex solid is just basic shapes hiding in plain sight. To solve for area or volume, you must first mentally "snap apart" the object into pieces you already know how to measure.

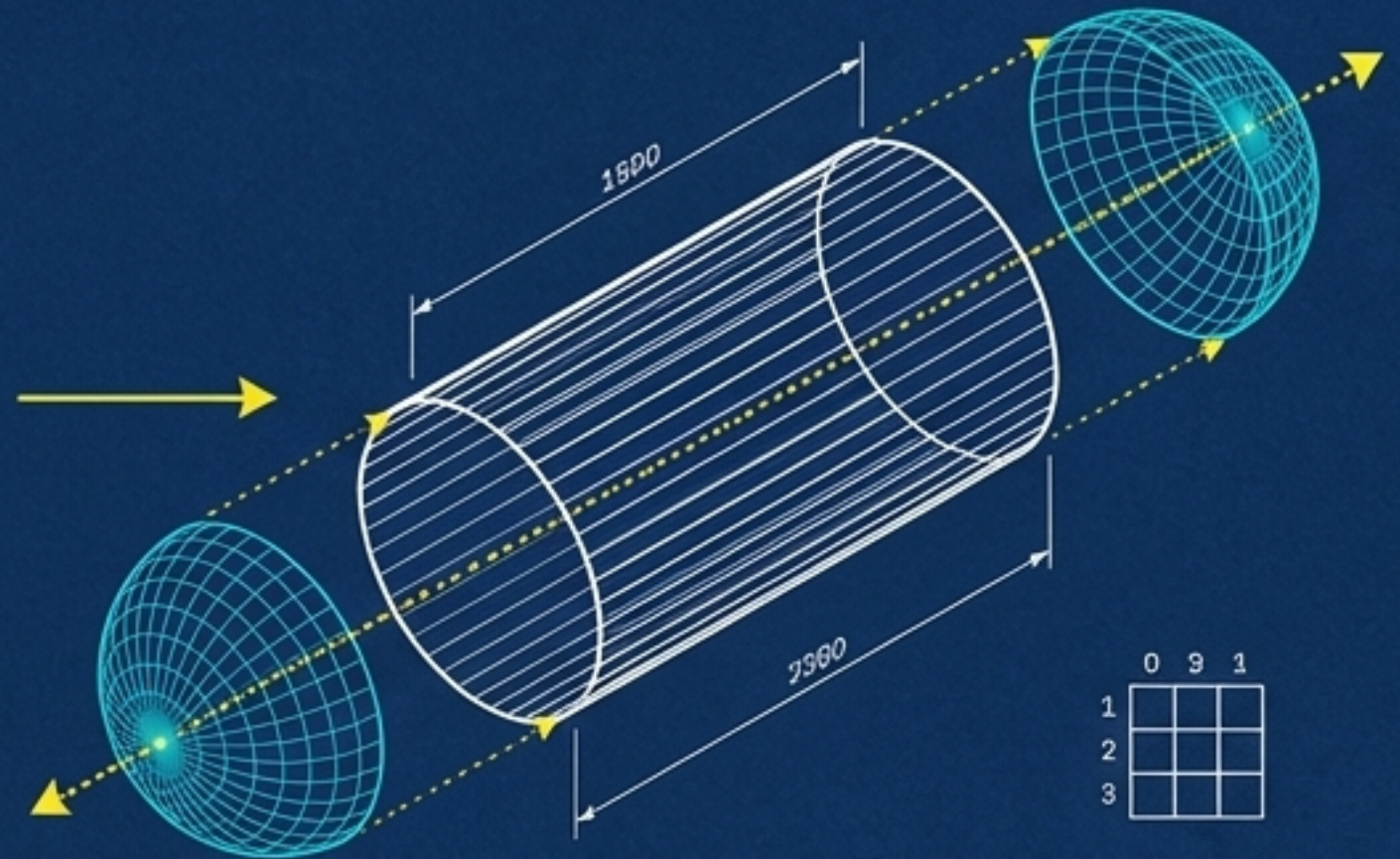
21.55



	0	3	1
1			
2			
3			

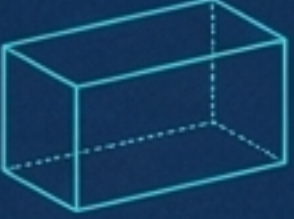

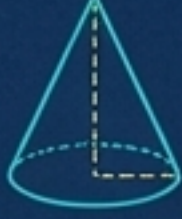




	0	3	1
1			
2			
3			

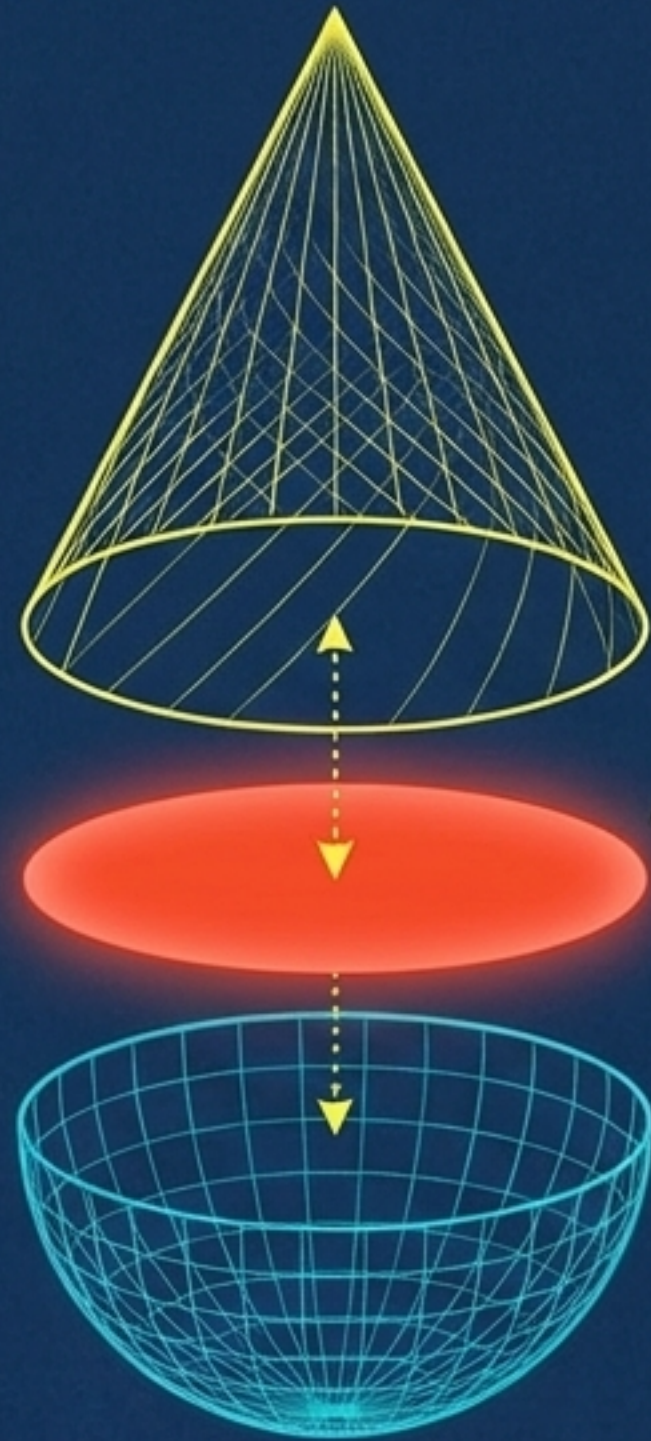


	0	3	1
1			
2			
3			

The Blueprint Toolkit

Shape	Curved Surface Area (CSA)	Total Surface Area (TSA)	Volume
Cuboid 	$2h(1+b)$	$2(1b+bh+h1)$	$1bh$
Cylinder 	$2\pi r h$	$2\pi r(r+h)$	$\pi r^2 h$
Cone 	$\pi r l$	$\pi r(r+l)$	$\frac{1}{3} \pi r^2 h$
Sphere 	$4\pi r^2$	$4\pi r^2$	$\frac{4}{3} \pi r^3$
 Hemisphere	$2\pi r^2$	$3\pi r^2$	$\frac{2}{3} \pi r^3$

The “Missing Faces” Rule of Surface Area



The Trap

$$\text{TSA}_{\text{new}} \neq \text{TSA}_{\text{cone}} + \text{TSA}_{\text{hemisphere}}$$

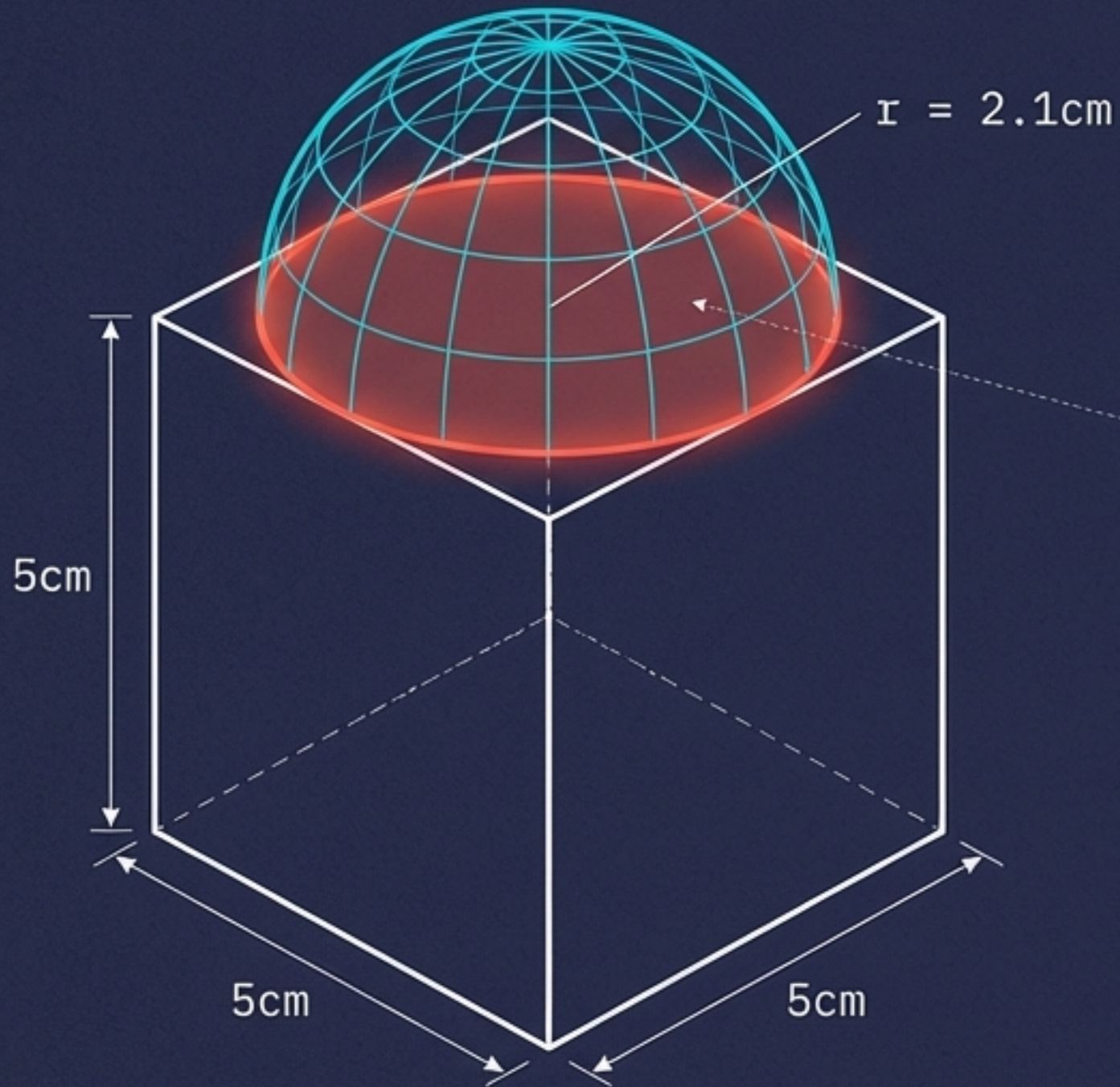
When shapes join, the faces that touch physically disappear into the interior.

The Rule

You must only calculate the EXPOSED outer skin.

$$\text{TSA}_{\text{toy}} = \text{CSA}_{\text{hemisphere}} + \text{CSA}_{\text{cone}}$$

Surface Area Example: The Overlap Subtraction



Total Surface Area of Cube:
 $6 \times (5\text{cm})^2 = 150 \text{ cm}^2$

MINUS



Subtract Base Area of Hemisphere:
 $-\pi r^2$

PLUS

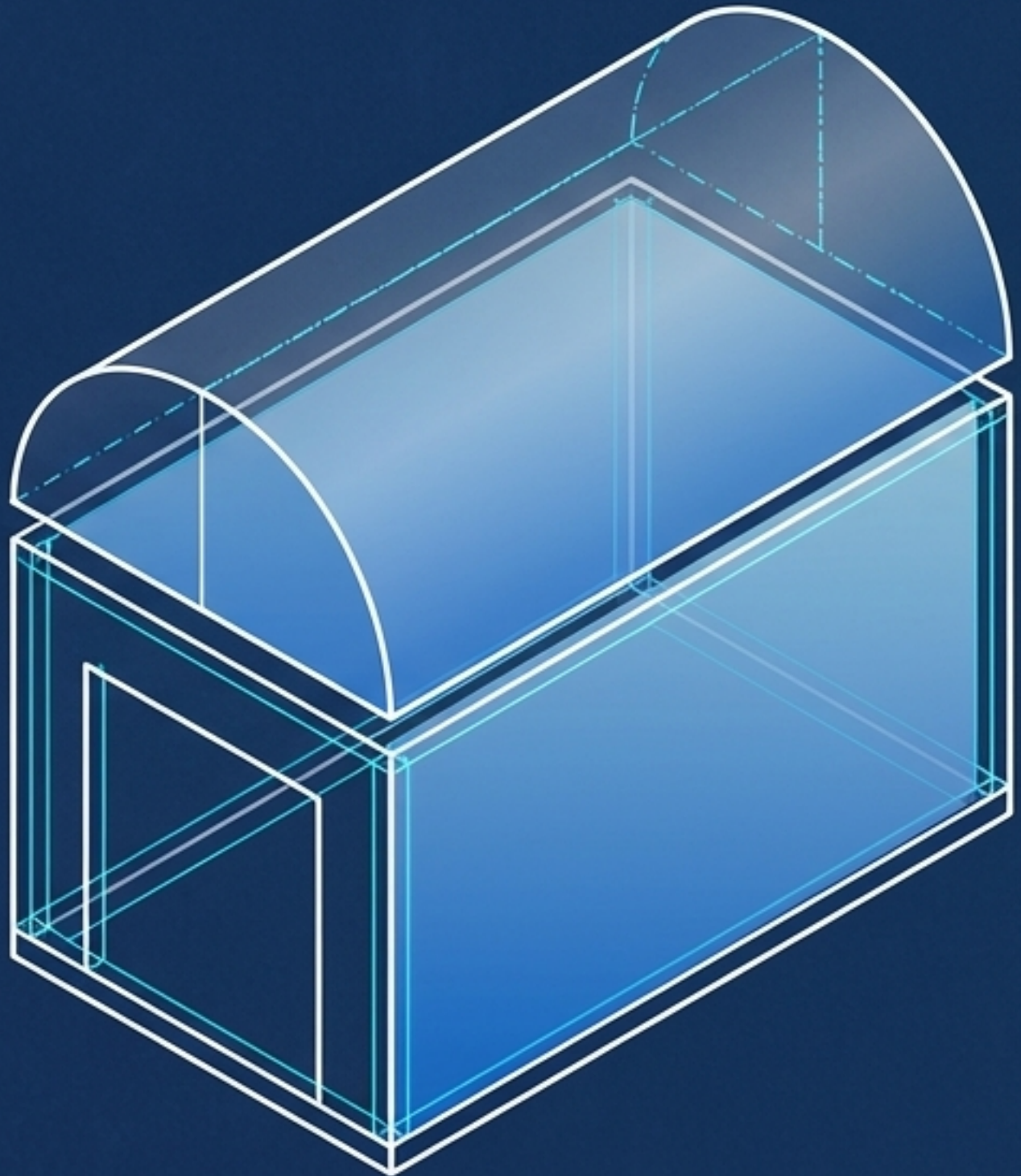


Add CSA of Hemisphere:
 $+2\pi r^2$

Final Calculation:

$$150 + \pi(2.1)^2 = 163.86 \text{ cm}^2$$

The Additive Rule of Volume



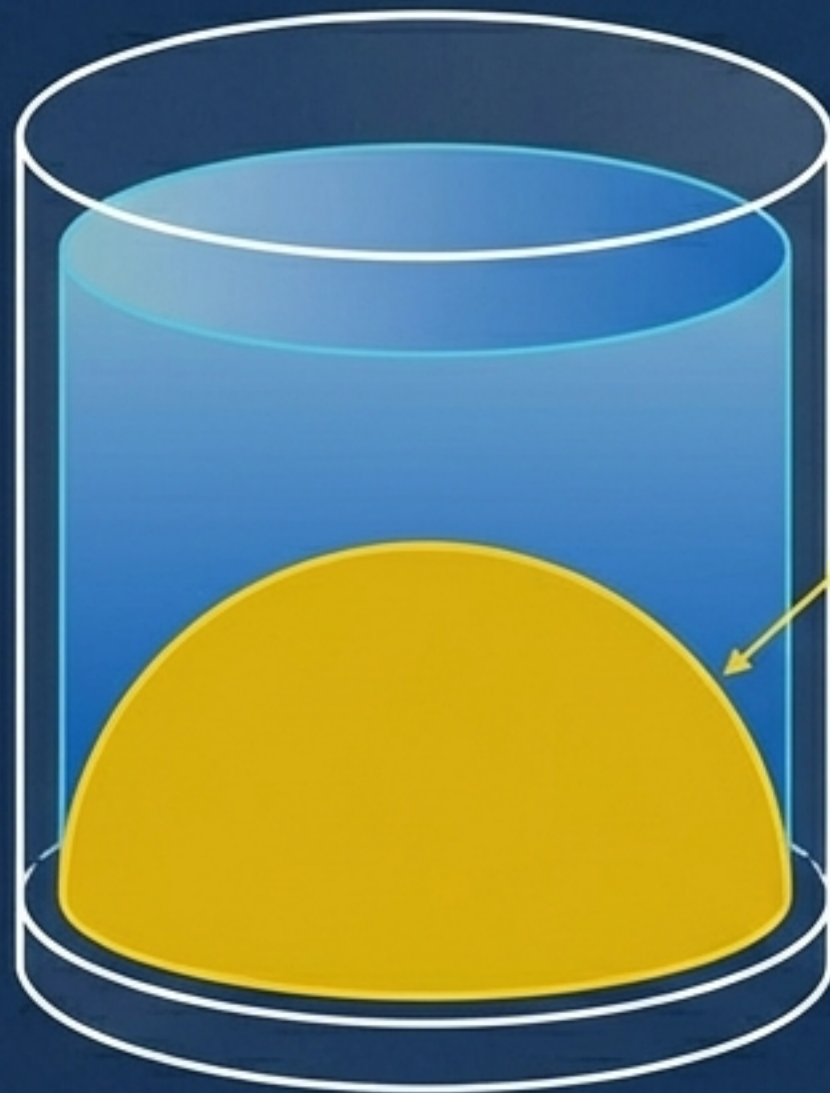
Unlike surface area, volume represents the raw, 3D space an object occupies. There are no “hidden faces” to subtract.

To find total capacity, simply add the volumes together.

$$\text{Volume}_{\text{total}} = \text{Volume}_{\text{cuboid}} + 1/2(\text{Volume}_{\text{cylinder}})$$
$$1128.75 \text{ m}^3 = (15 \times 7 \times 8) + 1/2(\pi \times 3.5^2 \times 15)$$

Volume Example: The Hidden Subtraction

Deceptive shapes require subtraction. The actual capacity is the apparent cylinder minus the intruding hemisphere.



← Apparent Capacity ($\pi r^2 h$):
196.25 cm³

← Minus Solid Hemisphere ($\frac{2}{3} \pi r^3$):
-32.71 cm³

Actual Capacity: 163.54 cm³

The Hollowing Paradox

(Adding)



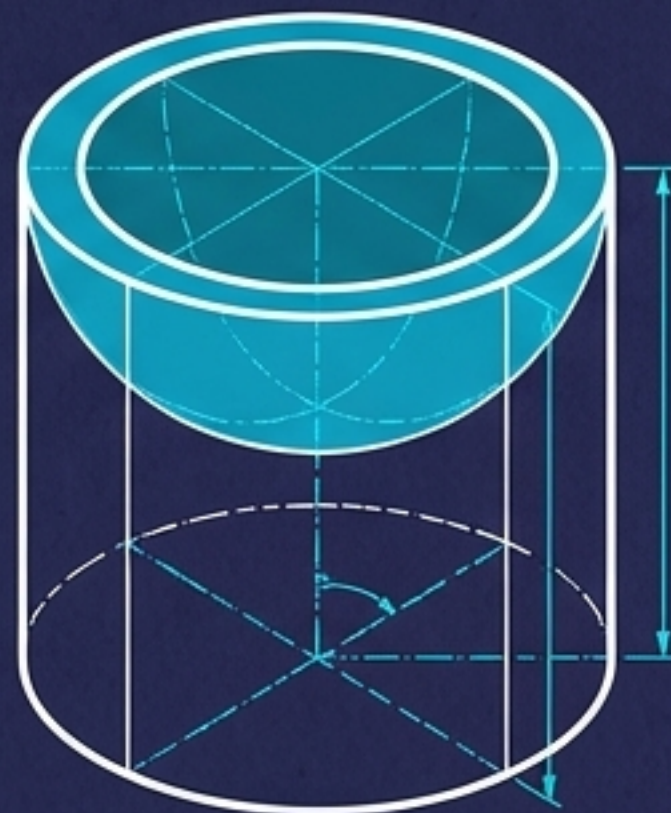
Volume



Surface Area



(Scooping)



Volume







Surface Area



The Paradox: Scooping out a cavity DECREASES Volume (you remove material), but INCREASES Total Surface Area (you expose new interior walls to the air).

Diagnostic Matrix: Area vs. Volume

 Surface Area (The “Skin”) 	 Volume (The “Space”) 
What parts can I physically touch or paint?	How much liquid or air can fit inside?
NEVER add Total Surface Areas (TSAs). Overlapping faces vanish.	ALWAYS add or subtract Total Volumes.
Add Curved Surface Areas (CSAs) + remaining flat bases.	$V_1 + V_2$ (if stacked) OR $V_1 - V_2$ (if hollowed out).

The 4-Step Master Algorithm

