

# Areas Related to Circles: Sectors & Segments

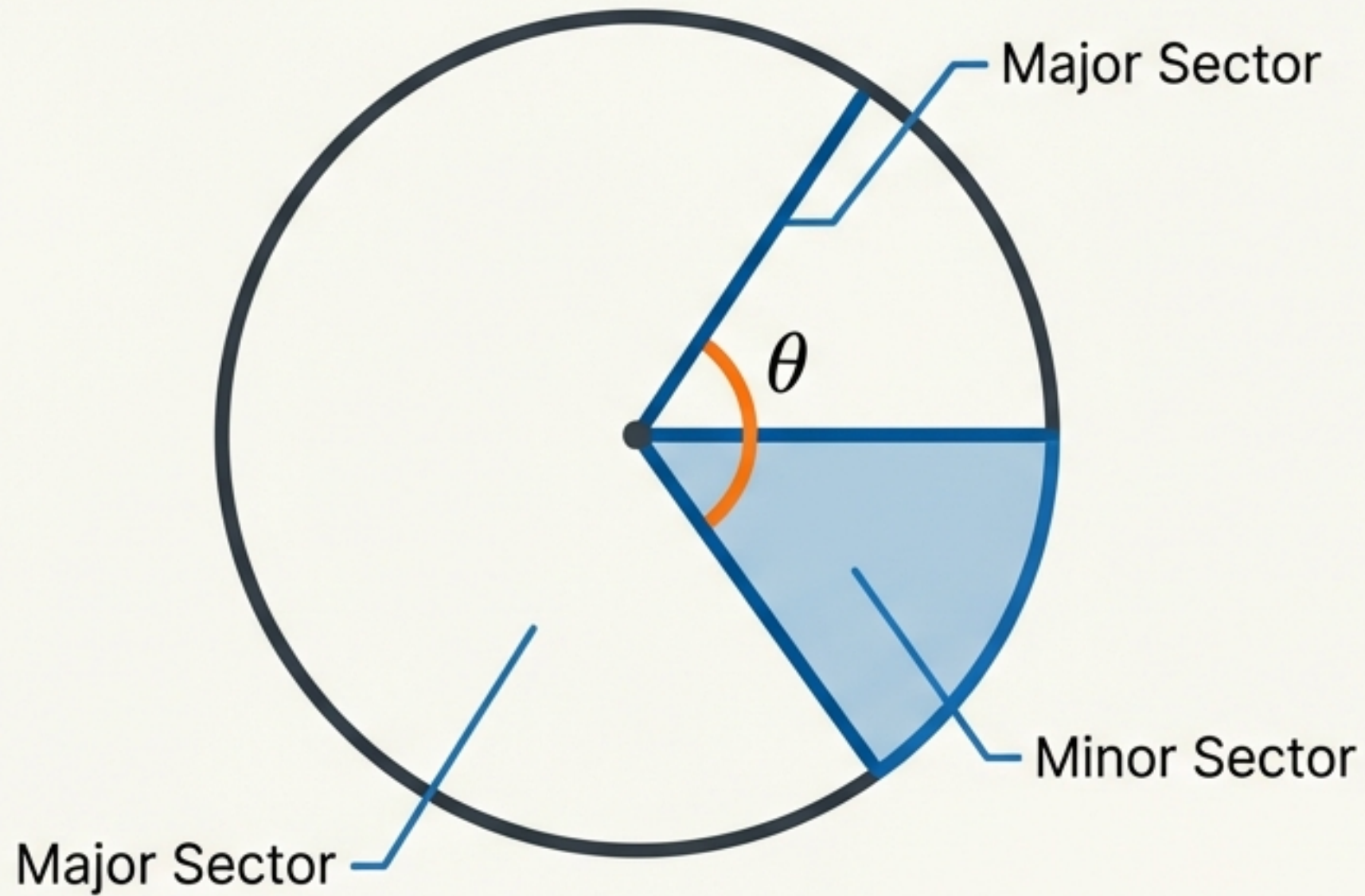
A Visual Toolkit for  
Geometric Problem Solving



# The Anatomy of a Circle

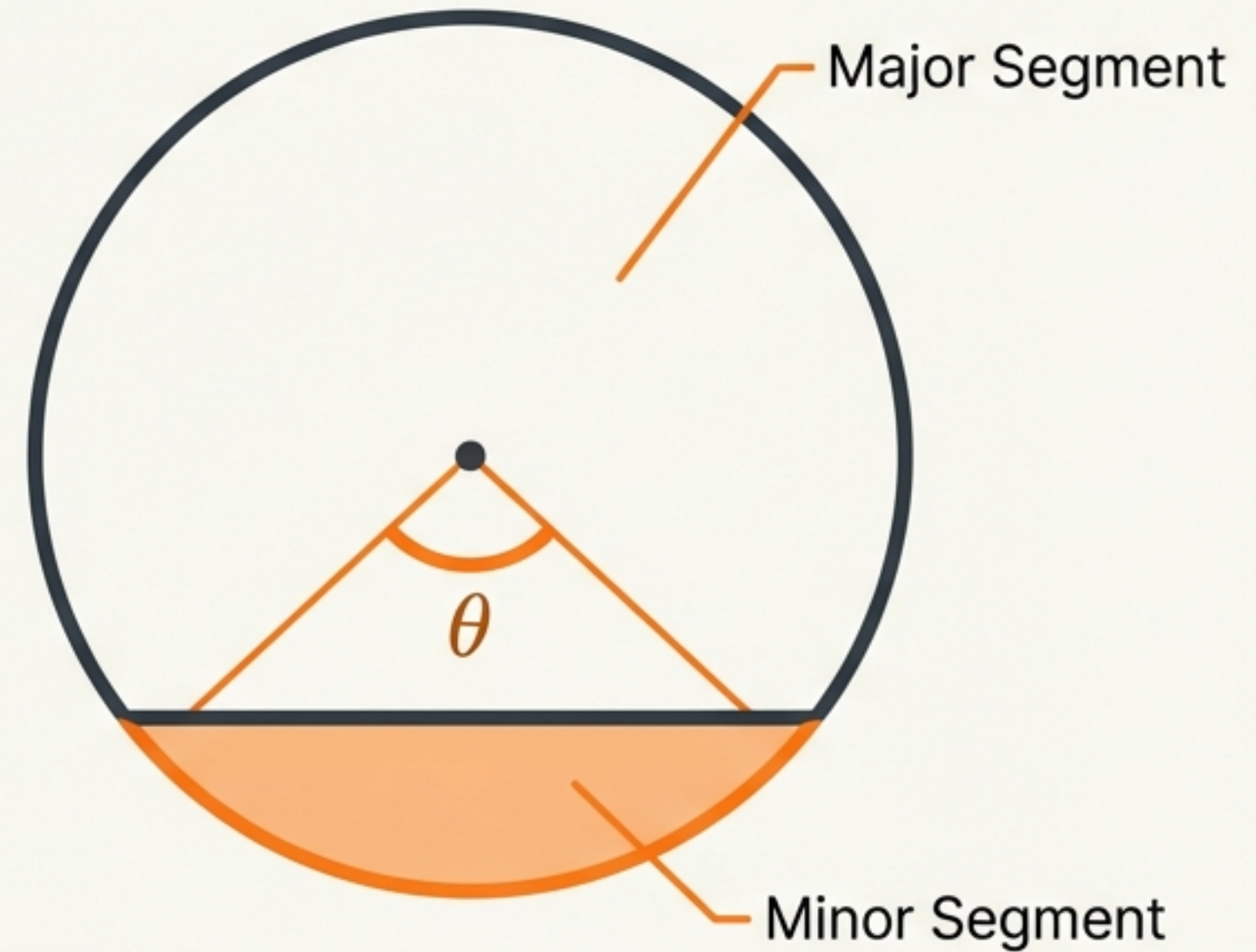
## The Sector

Bounded by two radii and an arc.



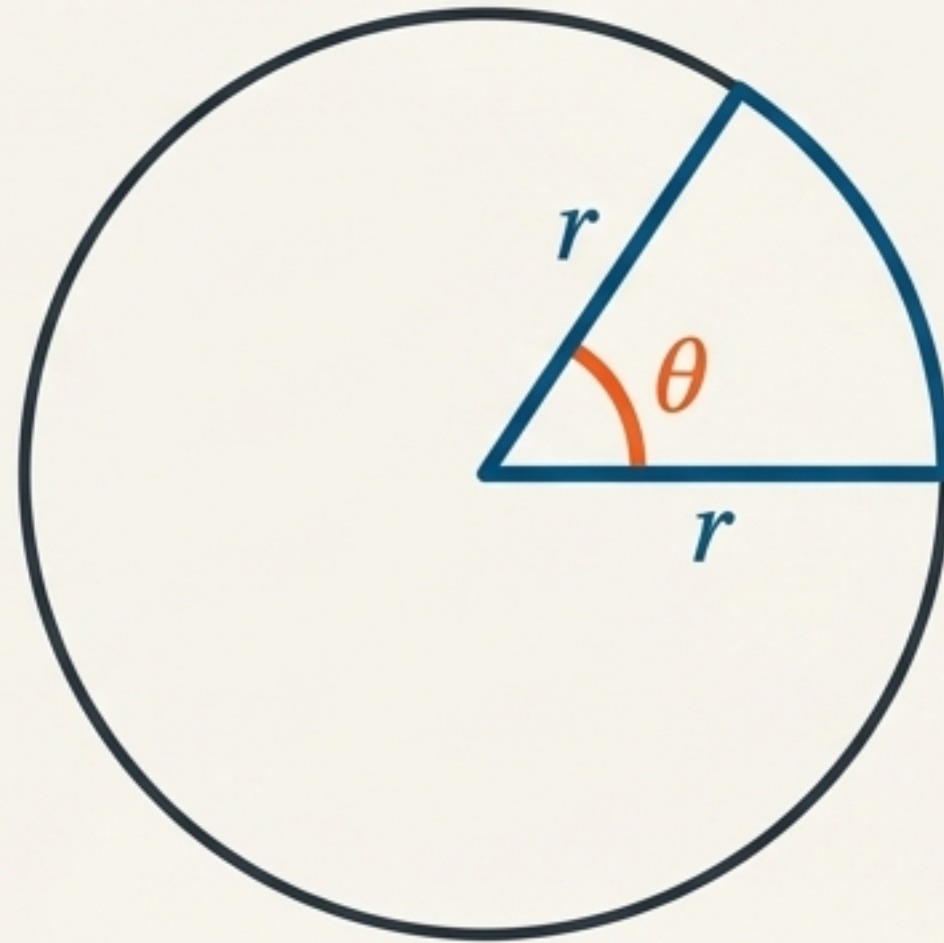
## The Segment

Bounded by a chord and an arc.



# The Sector Toolkit

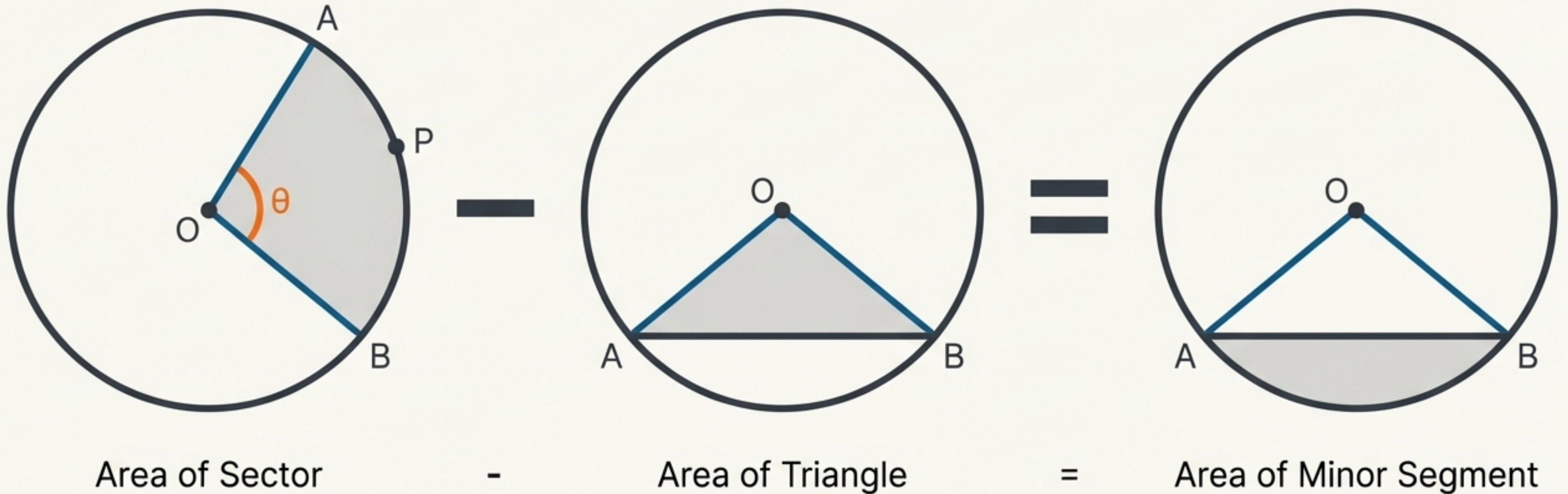
A full circle ( $360^\circ$ ) has an area of  $\pi r^2$  and a circumference of  $2\pi r$ . A sector is simply a fraction of that whole, defined by its angle ( $\theta$ ).



$$\text{Area of Sector} = \frac{\theta}{360} \times \pi r^2$$

$$\text{Length of Arc} = \frac{\theta}{360} \times 2\pi r$$

# The Segment Toolkit



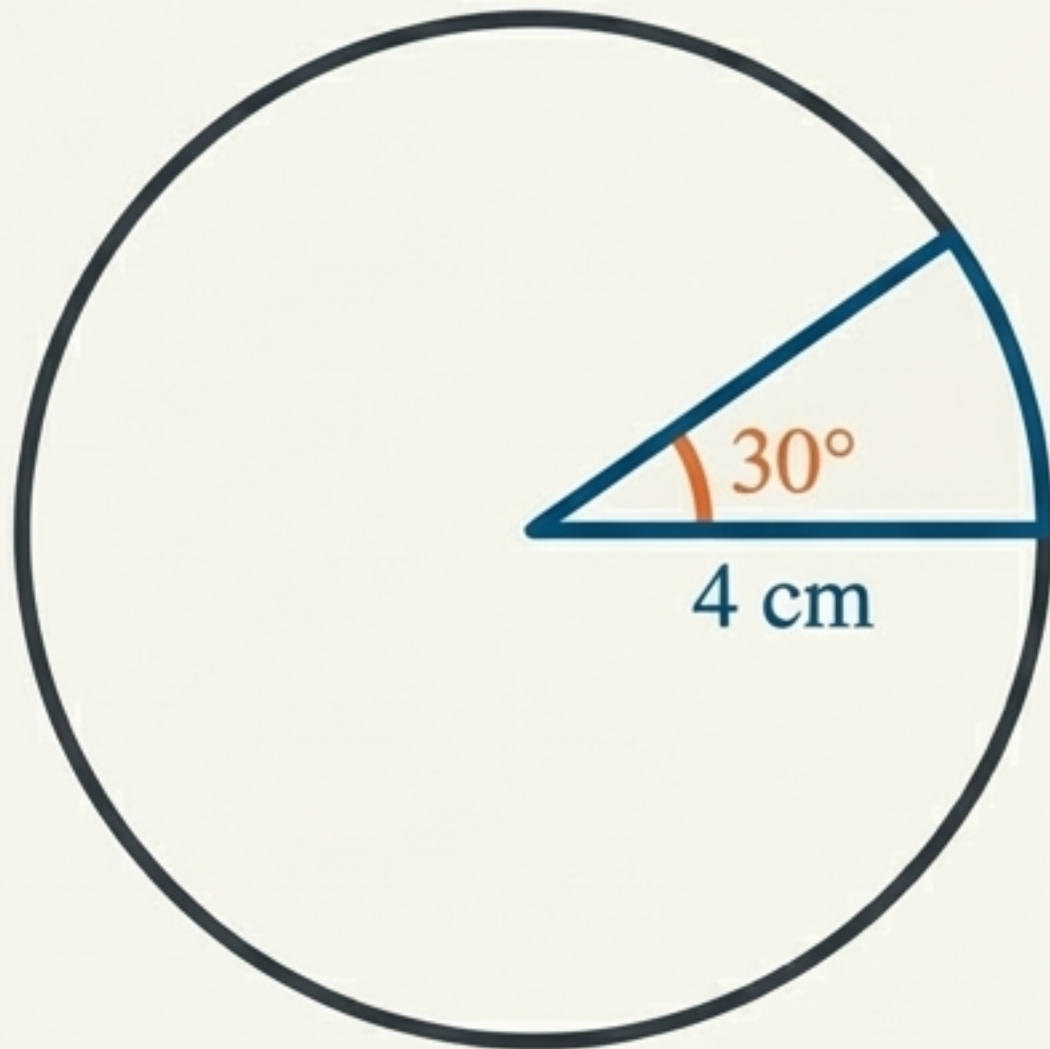
To find the Major Segment, simply subtract the Minor Segment from the total Area of the Circle ( $\pi r^2$ ).

# Working the Math: Sector Area

Radius ( $r$ ) = 4 cm

Angle ( $\theta$ ) =  $30^\circ$

(Use  $\pi = 3.14$ )



## Step 1: Minor Sector Area

$$\frac{30}{360} \times 3.14 \times 4^2$$

$$\frac{1}{12} \times 3.14 \times 16$$

Result: 4.19 cm<sup>2</sup> (approx.)

## Step 2: Major Sector Area (Two Paths)

Path A (Subtraction):

Total Circle Area ( $\pi r^2$ ) - Minor Sector

$$= (3.14 \times 16) - 4.19$$

$$= 46.1 \text{ cm}^2$$

Path B (Direct Formula):

Major Angle =  $360^\circ - 30^\circ = 330^\circ$

$$\frac{330}{360} \times 3.14 \times 4^2$$

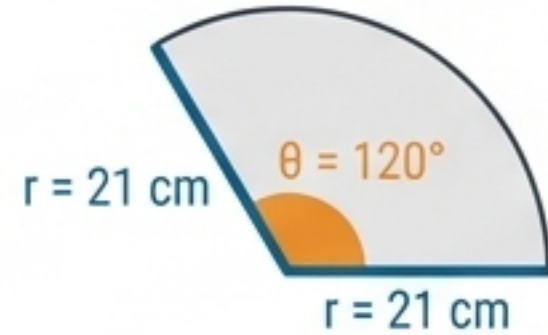
$$= 46.1 \text{ cm}^2$$

# Working the Math: Segment Area

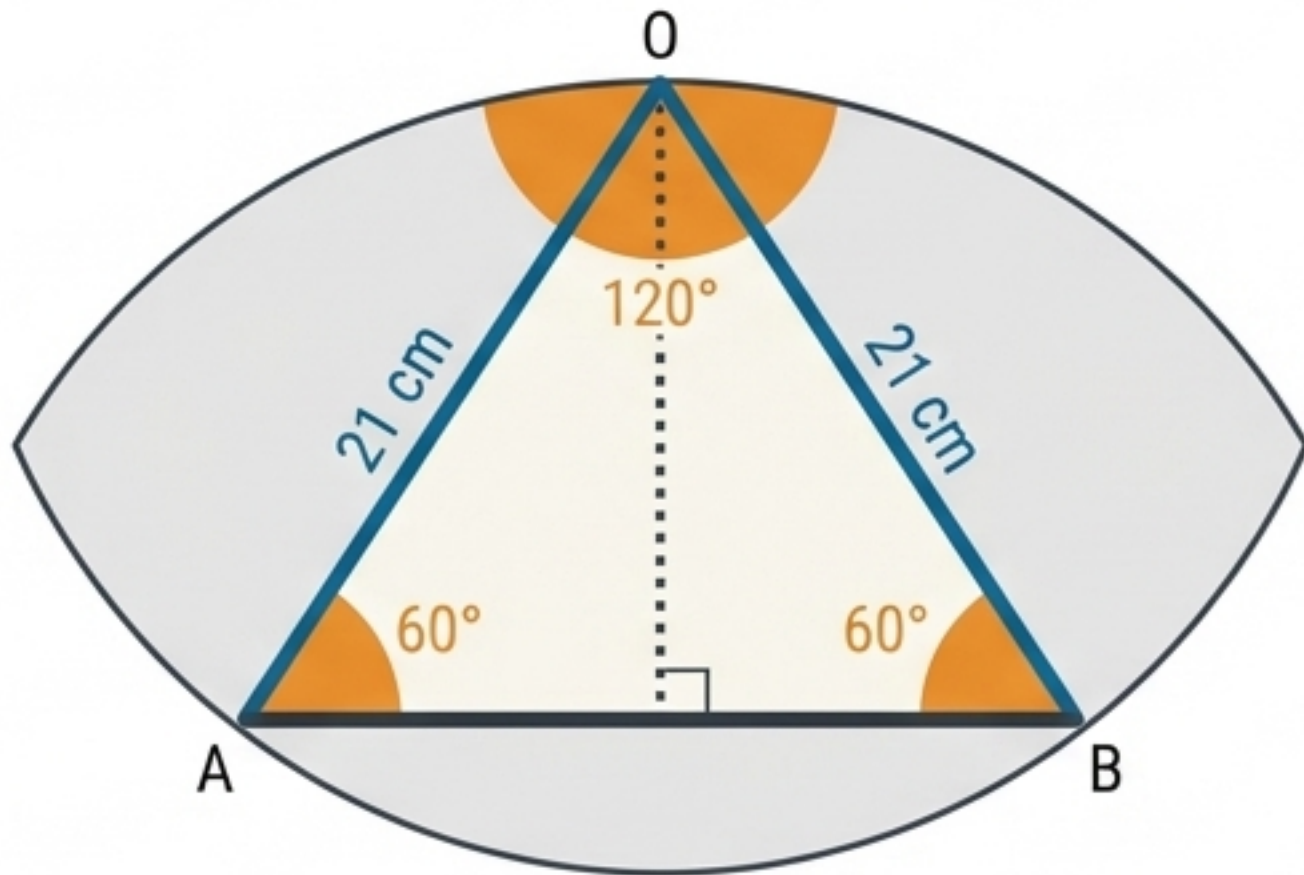
Radius ( $r$ ) = 21 cm | Angle ( $\theta$ ) =  $120^\circ$  | (Use  $\pi = 22/7$ )

## Step 1: The Sector

$$\begin{aligned} \text{Area} &= \frac{120}{360} \times \frac{22}{7} \times 21^2 \\ &= 462 \text{ cm}^2 \end{aligned}$$



## Step 2: The Triangle (The Trig Trick)



## Step 2: The Triangle (The Trig Trick)

Using  $\cos(60^\circ)$  and  $\sin(60^\circ)$ :

$$\begin{aligned} \text{Base} &= 21\sqrt{3} \text{ cm} \\ \text{Height} &= \frac{21}{2} \text{ cm} \\ \text{Area of Triangle} &= \frac{1}{2} \times \text{Base} \times \text{Height} \\ &= \frac{441\sqrt{3}}{4} \text{ cm}^2 \end{aligned}$$

Height =  $21 \times \cos(60^\circ)$

Base =  $2 \times (21 \times \sin(60^\circ))$

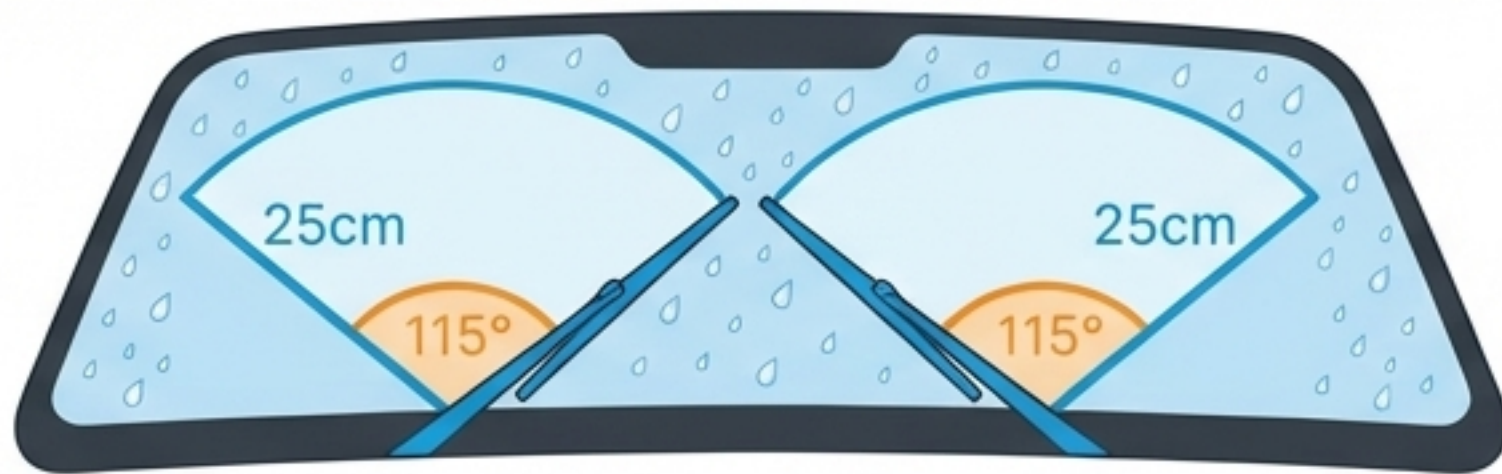
A diagram of a triangle with vertices O (top), A (bottom left), and B (bottom right). The sides OA and OB are labeled 21 cm. The angles at A and B are labeled 60 degrees. A dashed vertical line from O to the base AB is labeled as the height. A horizontal line from O perpendicular to the height is labeled as the base. The area between the triangle and the arc is shaded light blue.

## Step 3: The Subtraction

$$\begin{aligned} \text{Segment Area} &= \text{Sector Area} - \text{Triangle Area} \\ &= 462 - \frac{441\sqrt{3}}{4} \text{ cm}^2 \end{aligned}$$

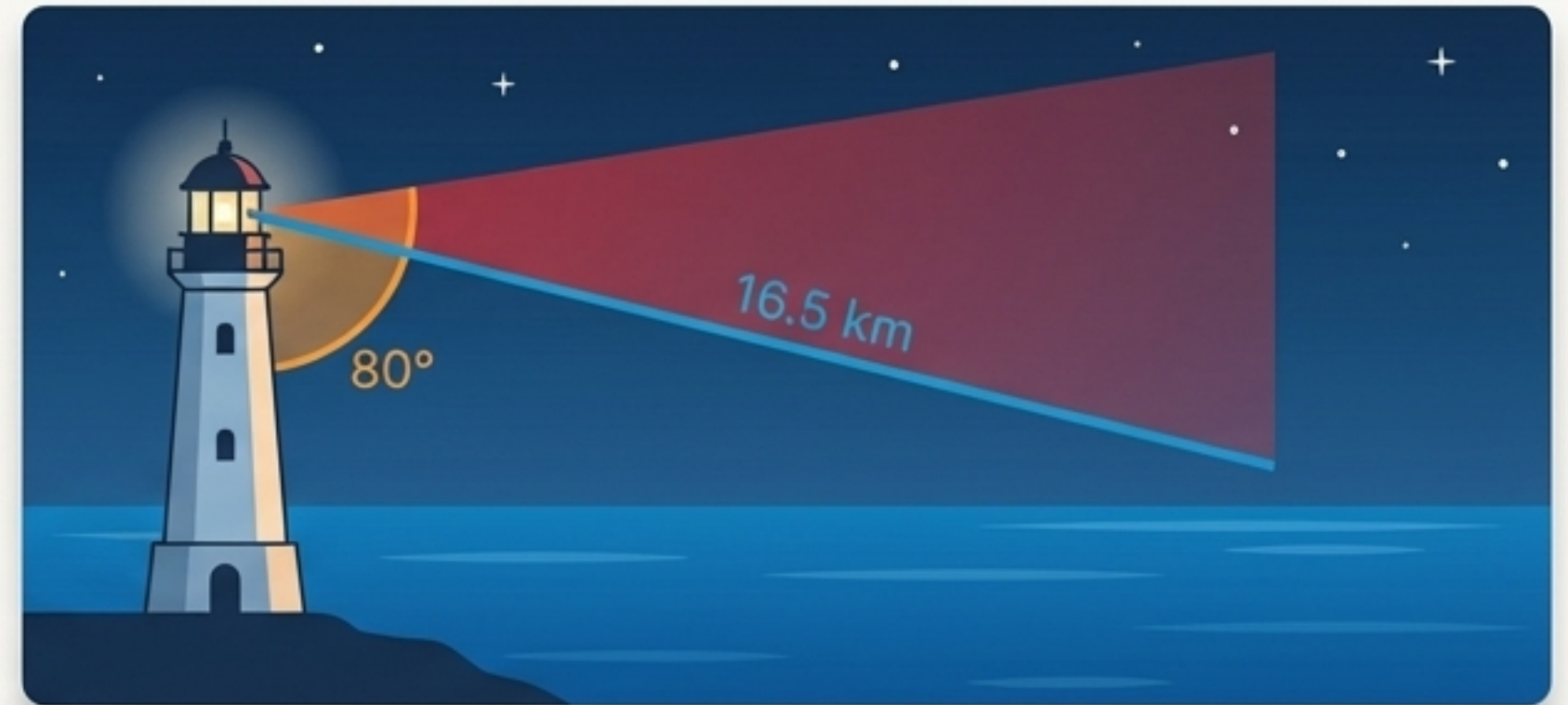


# Geometry in the Real World



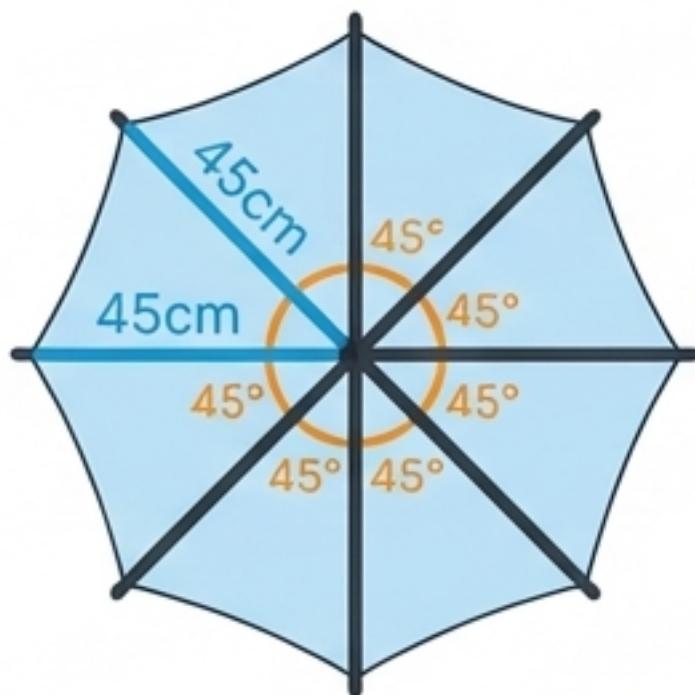
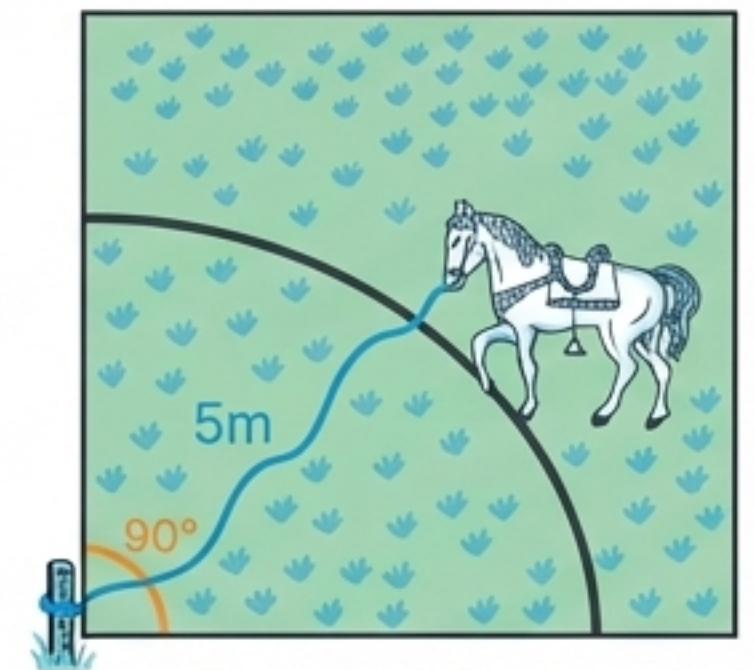
## Automotive

Wipers with 25cm blades sweeping  $115^\circ$  angles. Total area cleaned requires sector math.



## Agriculture

A horse on a 5m rope at the corner of a square field creates a perfect  $90^\circ$  quadrant sector of grazing area.



## Product Design

8 equally spaced ribs acting as radii (45cm) to create eight perfect  $45^\circ$  sectors.

# The Master Cheat Sheet

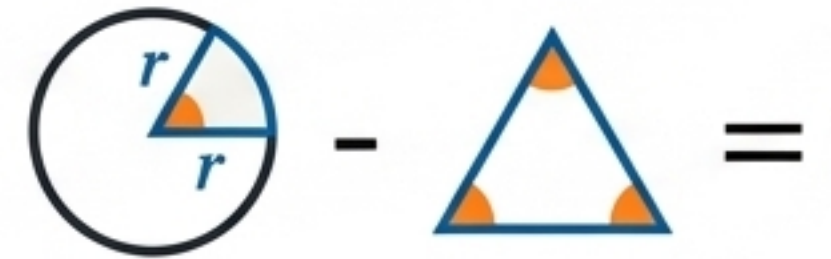
## Arc Length

$$\left(\frac{\theta}{360}\right) \times 2\pi r$$

## Sector Area

$$\left(\frac{\theta}{360}\right) \times \pi r^2$$

## Segment Area



Sector Area - Triangle Area

**The Golden Rule:** Major Region Area = Total Area of Circle ( $\pi r^2$ ) - Minor Region Area.