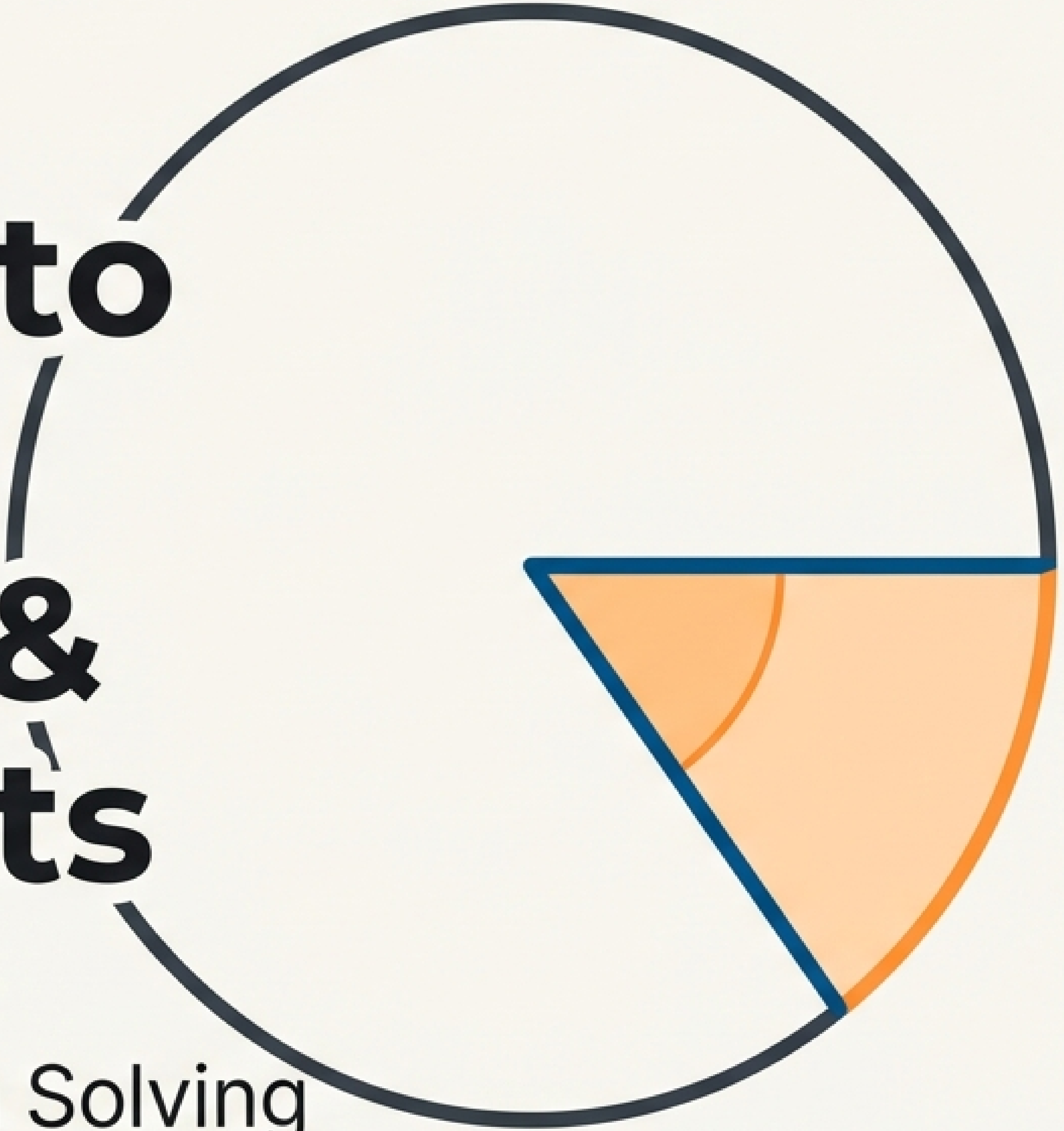
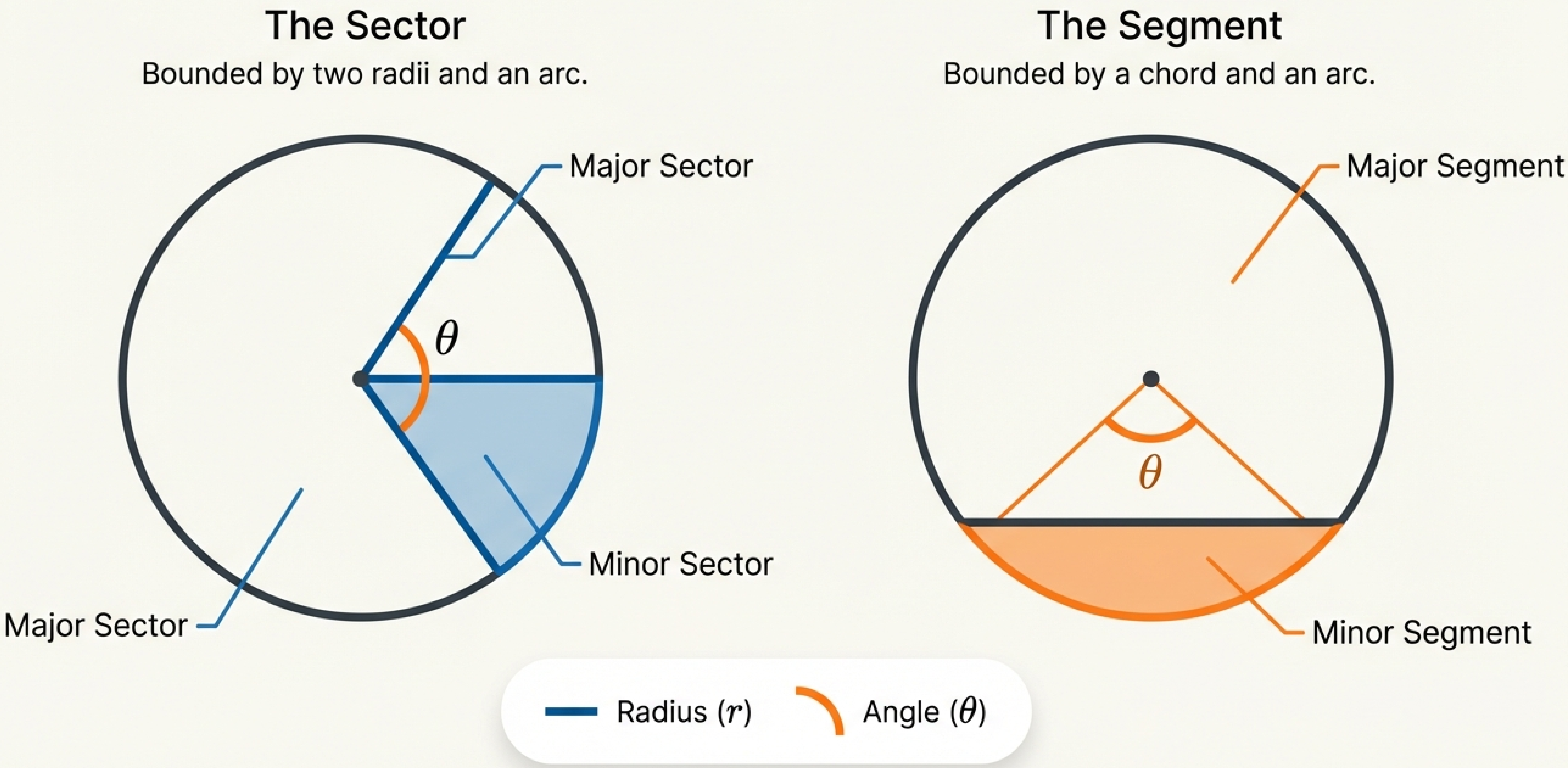
Areas Related to Circles: Sectors & Segments

A Visual Toolkit for Geometric Problem Solving



The Anatomy of a Circle



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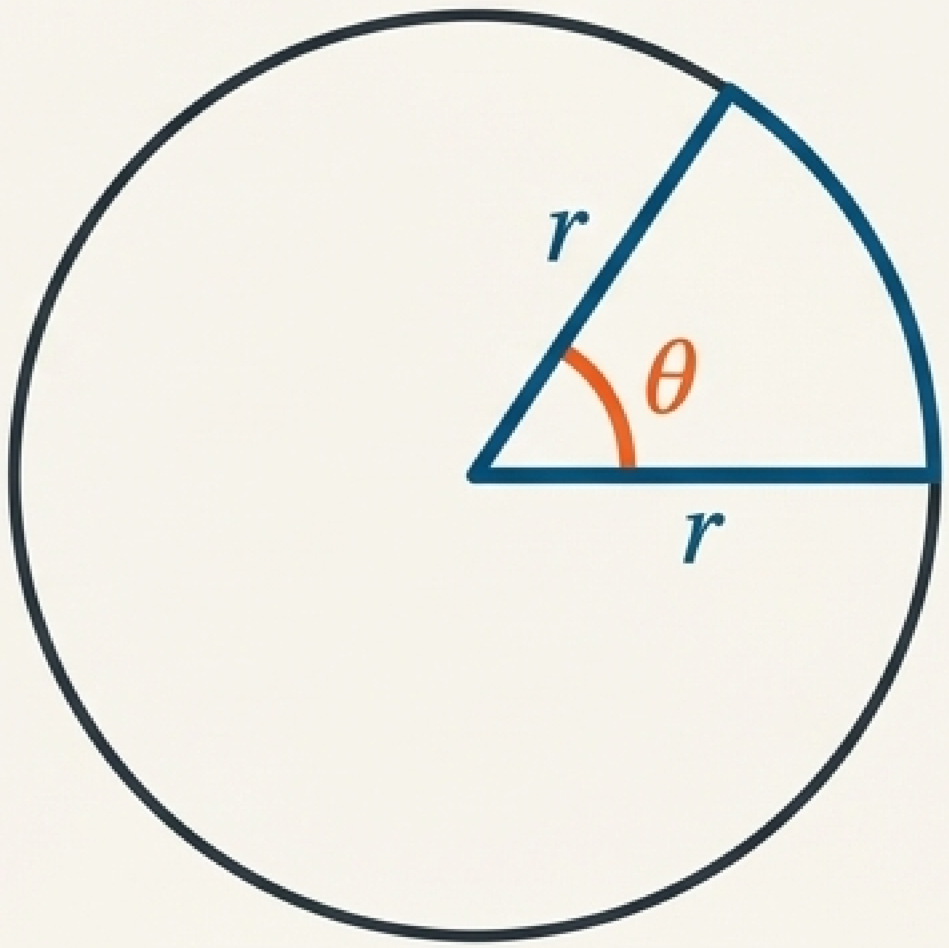
The Sector Toolkit

A full circle (360°) has an area of πr^2 and a circumference of $2\pi r$. A sector is simply a fraction of that whole, defined by its angle (θ).



$$\text{Area of Sector} = \frac{\theta}{360} \times \pi r^2$$

$$\text{Length of Arc} = \frac{\theta}{360} \times 2\pi r$$

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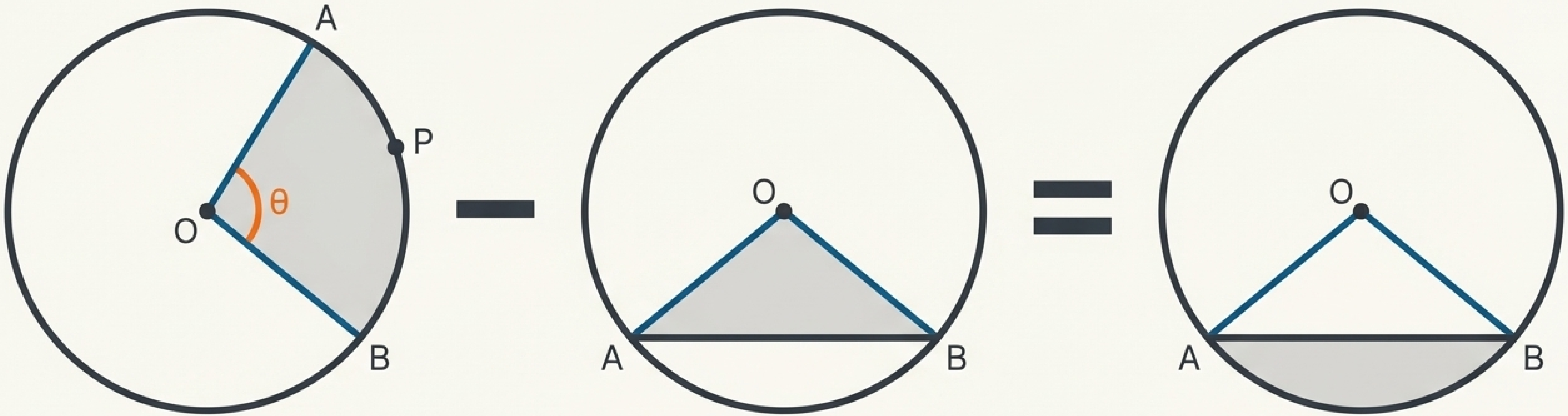
The Segment Toolkit

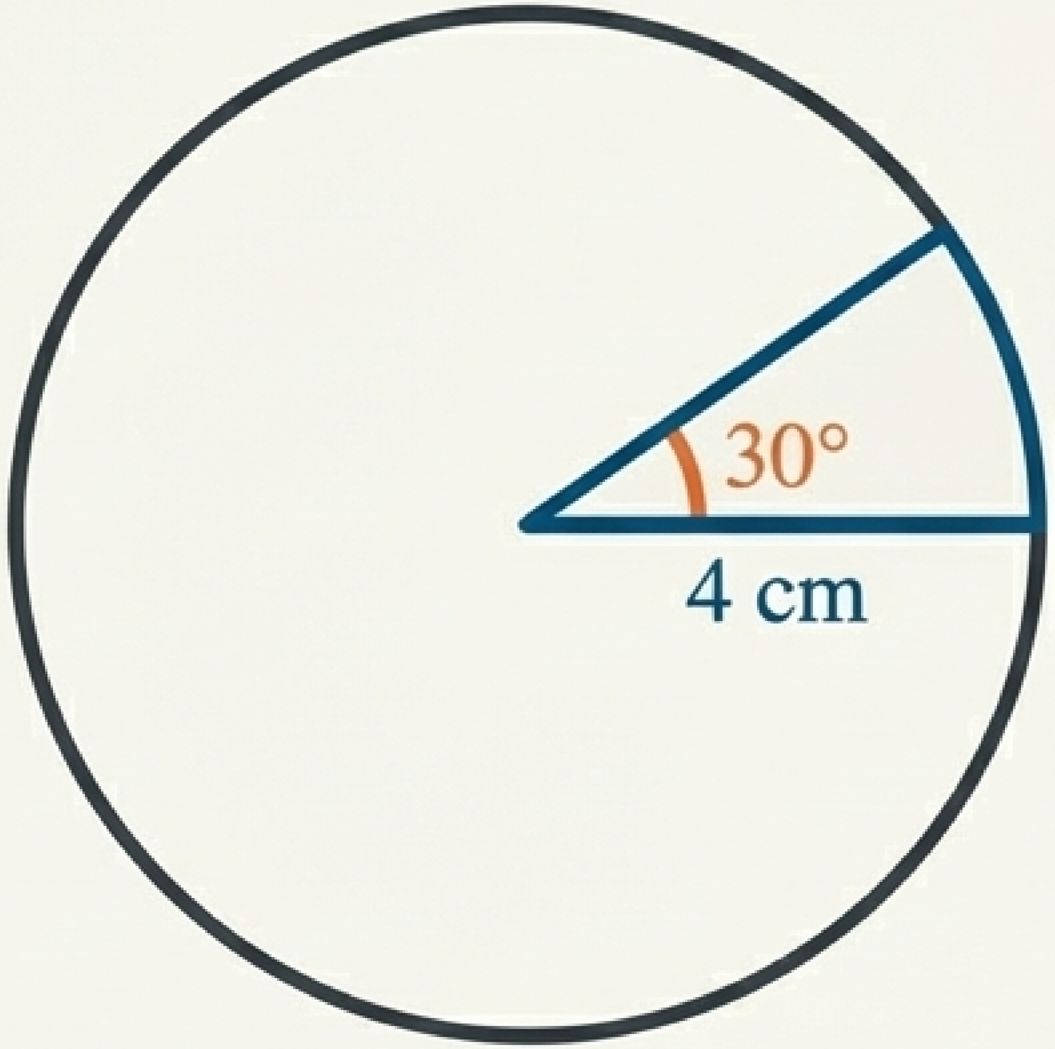


Area of Sector - Area of Triangle = Area of Minor Segment

To find the Major Segment, simply subtract the Minor Segment from the total Area of the Circle (πr^2).

Working the Math: Sector Area

Radius (r) = 4 cm
Angle (θ) = 30°
(Use π = 3.14)



Step 1: Minor Sector Area

$$\frac{30}{360} \times 3.14 \times 4^2$$

$$\frac{1}{12} \times 3.14 \times 16$$

Result: 4.19 cm^2 (approx.)

Step 2: Major Sector Area (Two Paths)

Path A (Subtraction):

Total Circle Area (πr^2) - Minor Sector

$= (3.14 \times 16) - 4.19$

$= 46.1\ cm^2$

Path B (Direct Formula):

Major Angle = 360° - 30° = 330°

$$\frac{330}{360} \times 3.14 \times 4^2$$

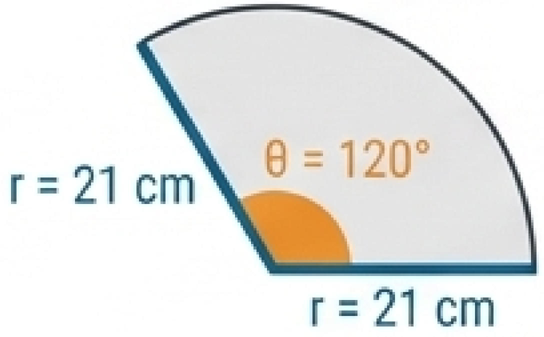
$= 46.1\ cm^2$

Working the Math: Segment Area

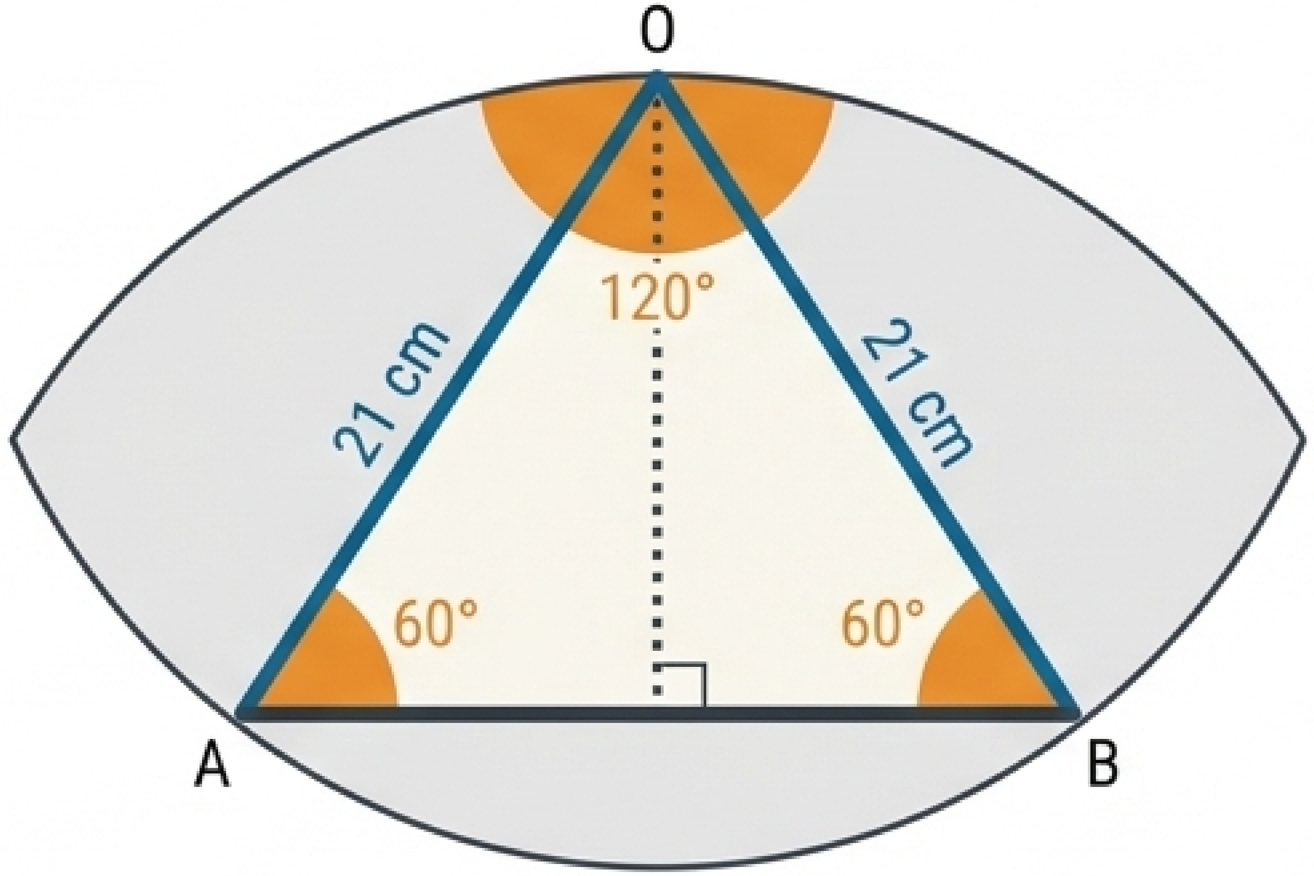
Radius (r) = 21 cm | Angle (θ) = 120° | (Use π = 22/7)

Step 1: The Sector

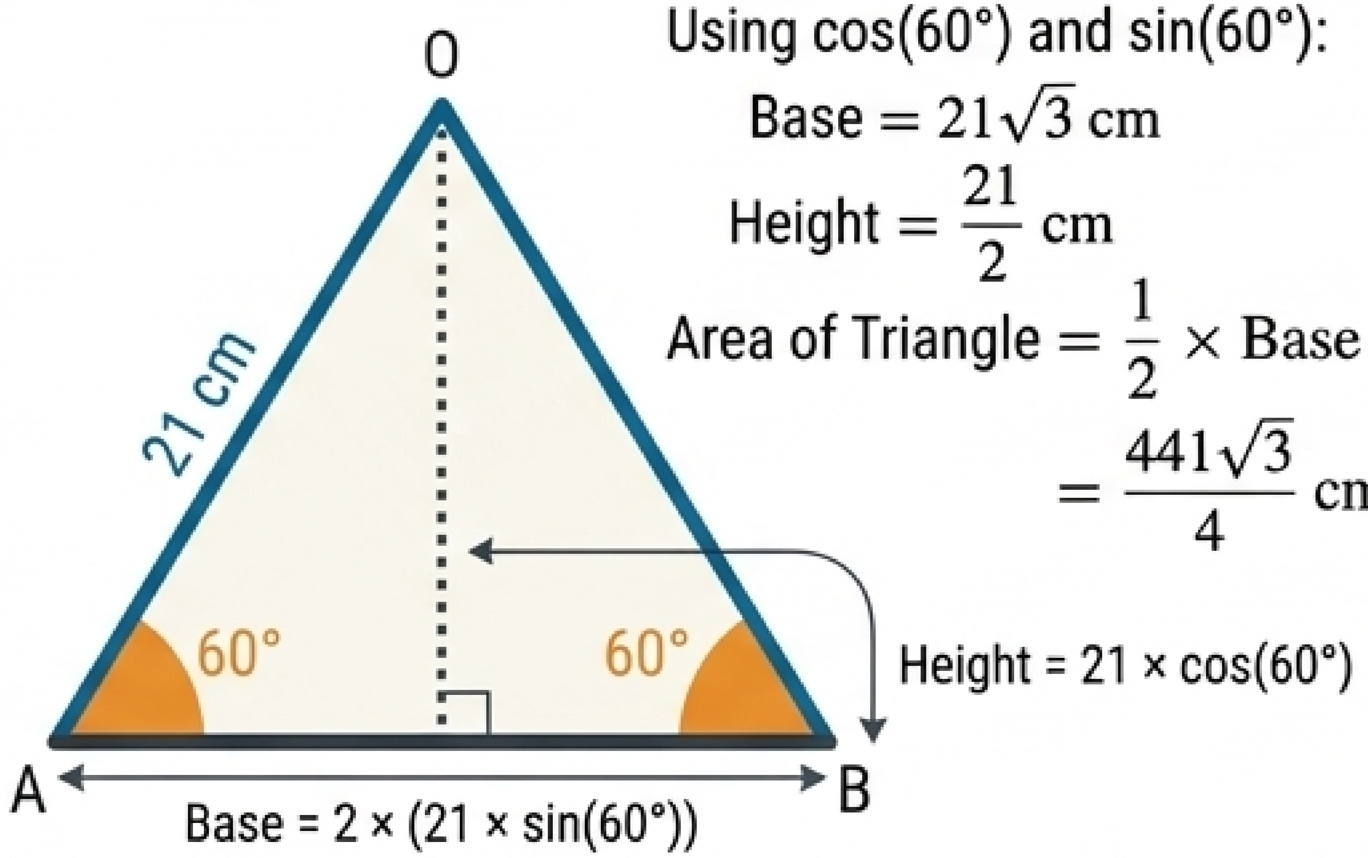
$$\text{Area} = \frac{120}{360} \times \frac{22}{7} \times 21^2$$
$$= 462 \text{ cm}^2$$



Step 2: The Triangle (The Trig Trick)



Step 2: The Triangle (The Trig Trick)

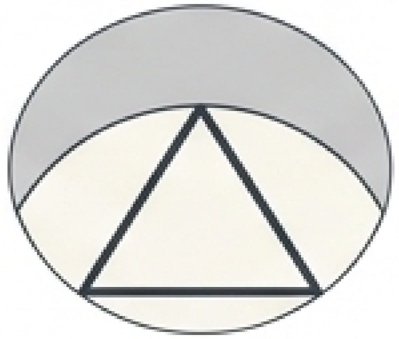


Using cos(60°) and sin(60°):

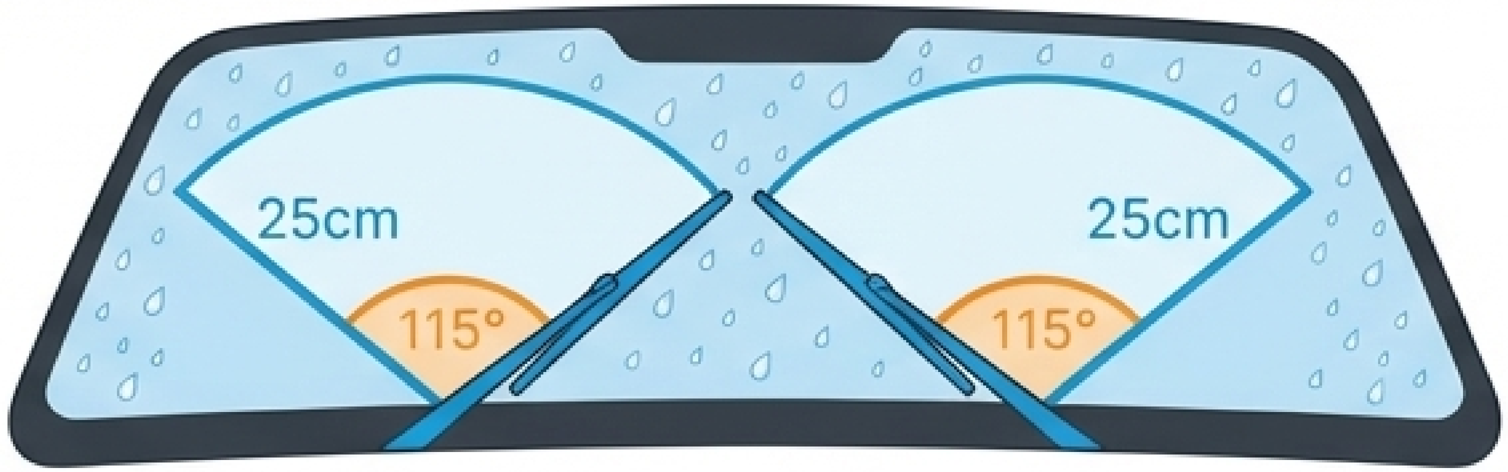
$$\text{Base} = 21\sqrt{3} \text{ cm}$$
$$\text{Height} = \frac{21}{2} \text{ cm}$$
$$\text{Area of Triangle} = \frac{1}{2} \times \text{Base} \times \text{Height}$$
$$= \frac{441\sqrt{3}}{4} \text{ cm}^2$$

Step 3: The Subtraction

$$\text{Segment Area} = \text{Sector Area} - \text{Triangle Area}$$
$$= 462 - \frac{441\sqrt{3}}{4} \text{ cm}^2$$

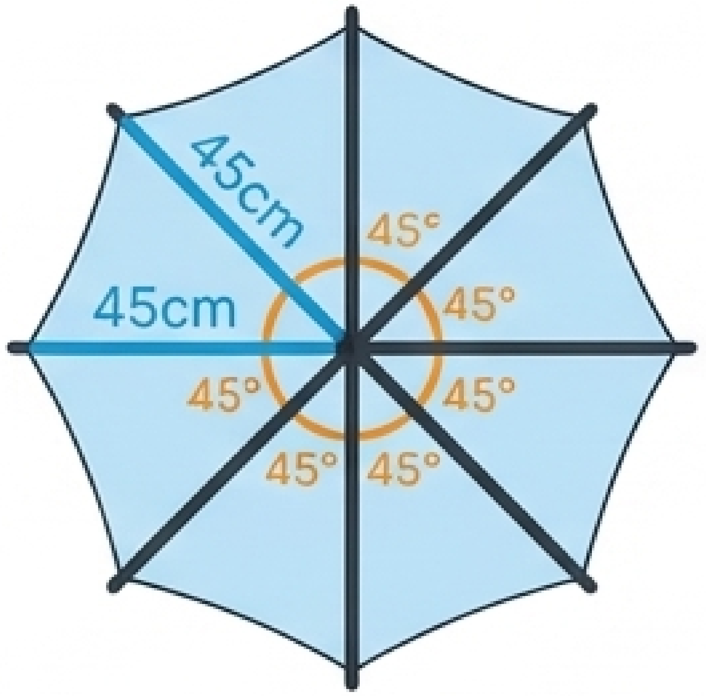


Geometry in the Real World



Automotive

Wipers with 25cm blades sweeping 115° angles. Total area cleaned requires sector math.

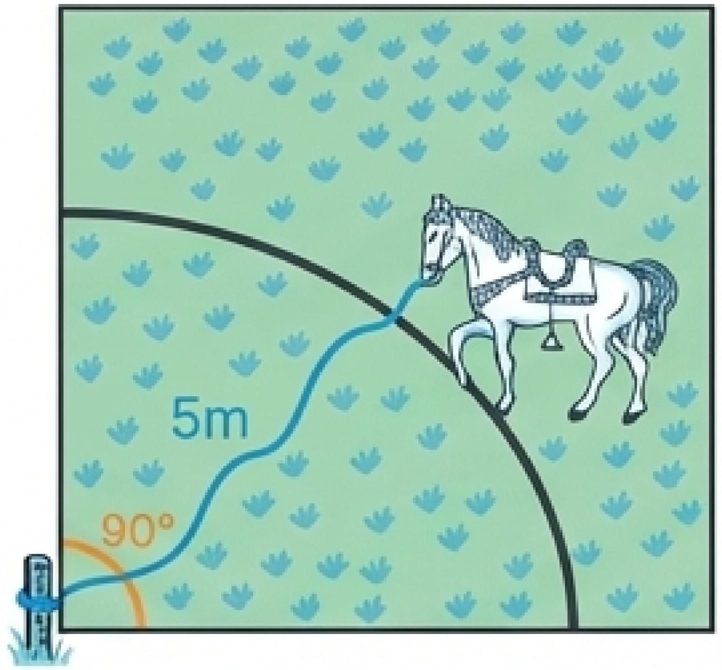


Product Design

8 equally spaced ribs acting as radii (45cm) to create eight perfect 45° sectors.

Agriculture

A horse on a 5m rope at the corner of a square field creates a perfect 90° quadrant sector of grazing area.



The Master Cheat Sheet

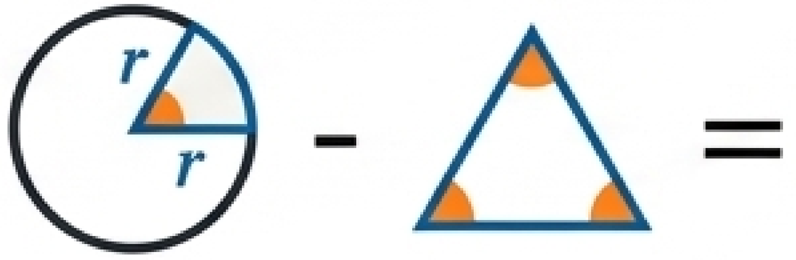
Arc Length

$$\left(\frac{\theta}{360}\right) \times 2\pi r$$

Sector Area

$$\left(\frac{\theta}{360}\right) \times \pi r^2$$

Segment Area



Sector Area - Triangle Area

The Golden Rule: Major Region Area = Total Area of Circle (πr^2) - Minor Region Area.