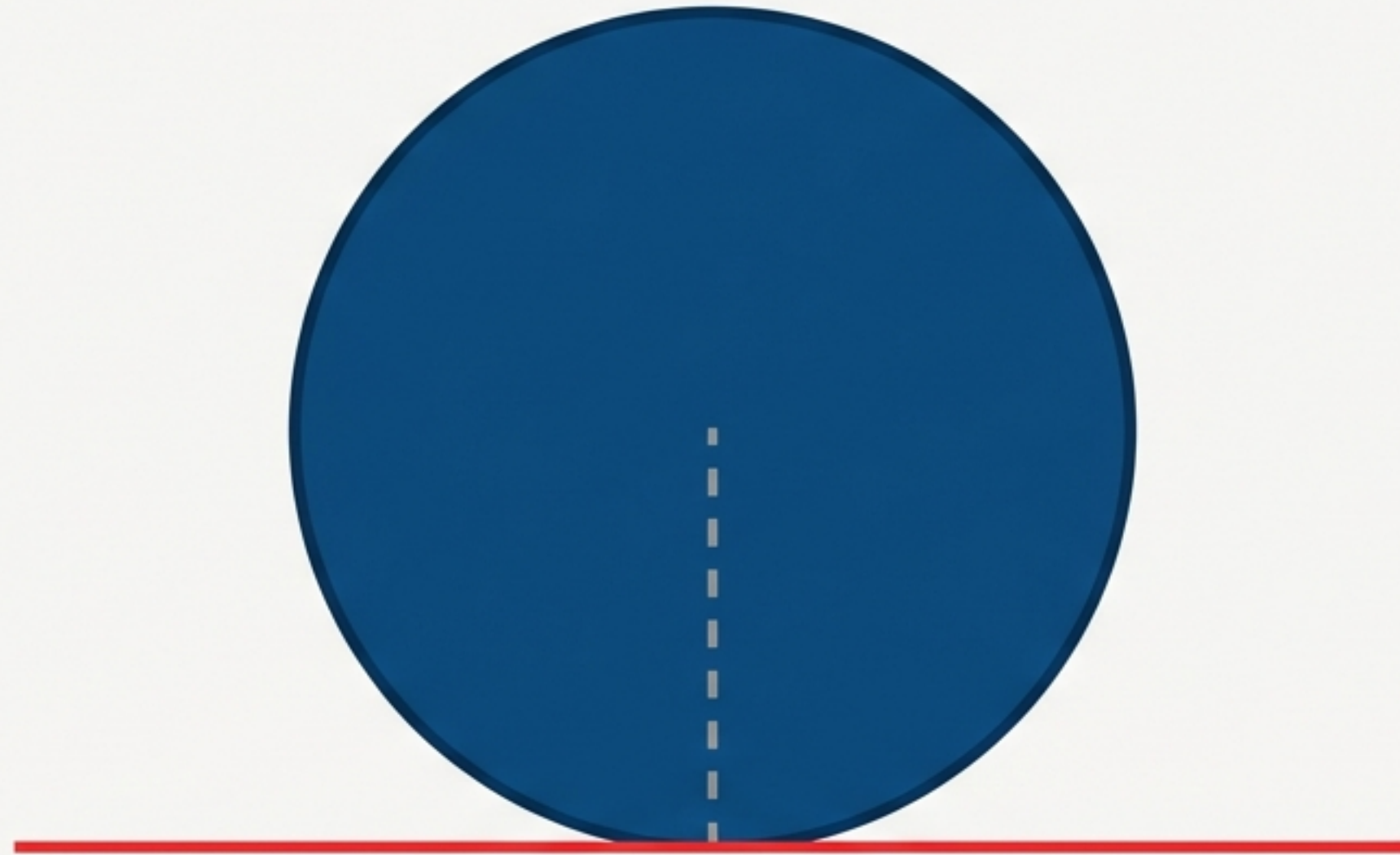
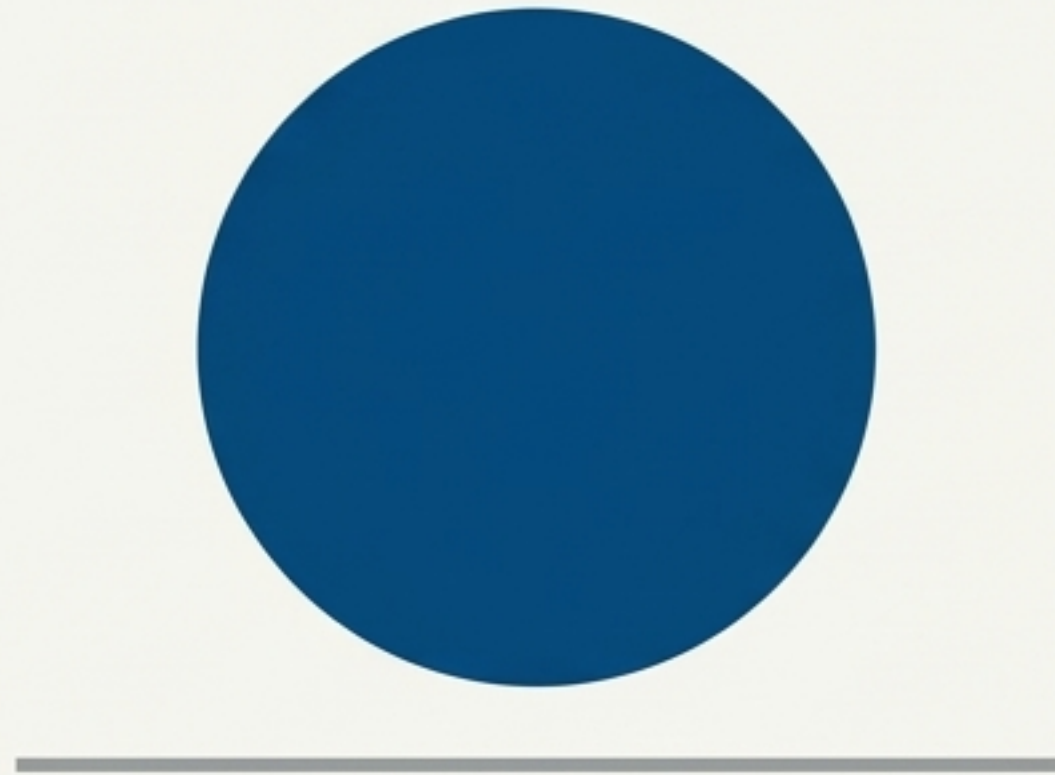


Circles & Tangents

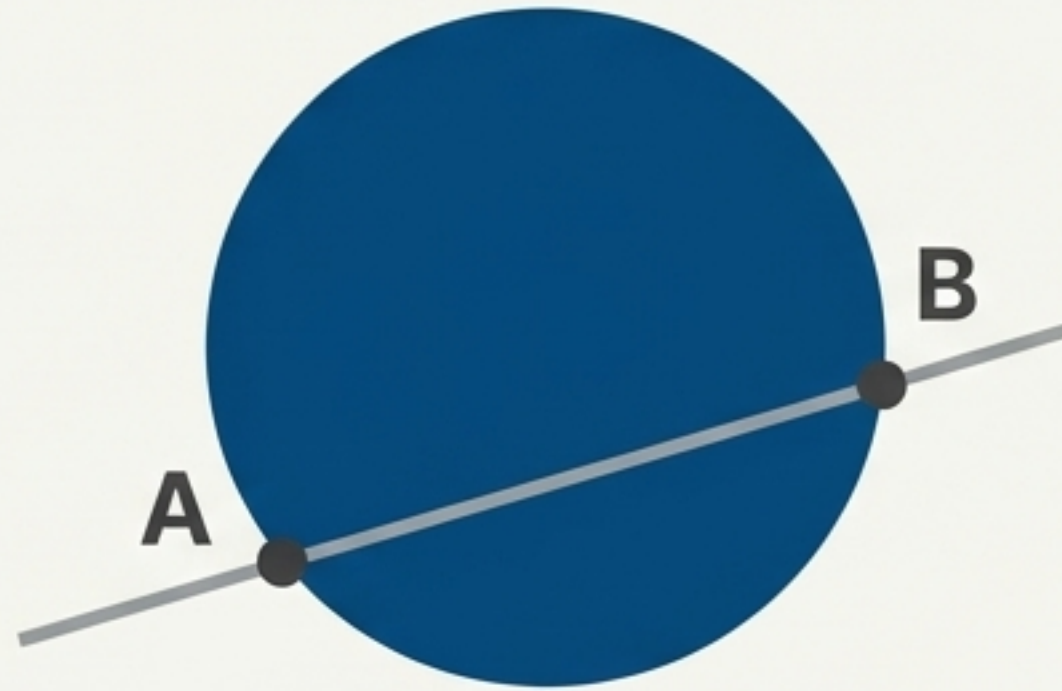
Unlocking Geometric Properties



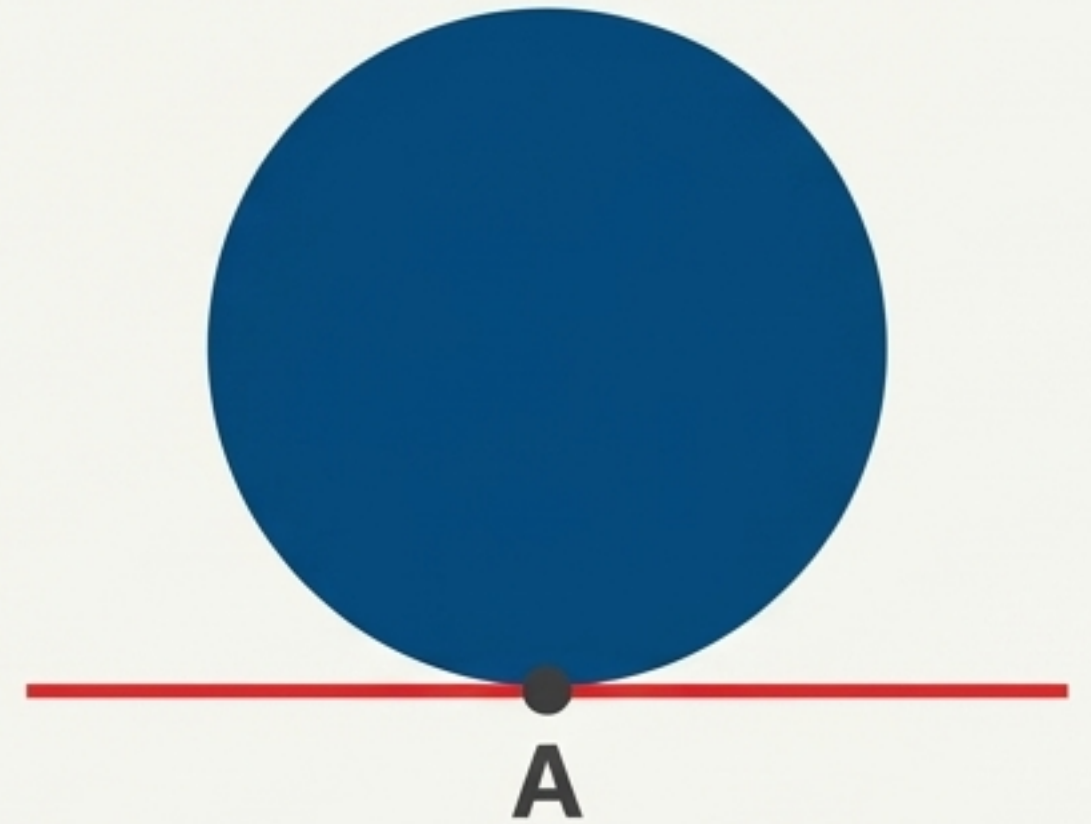
Three Ways to Cross a Circle



Non-Intersecting Line
0 Points of Contact

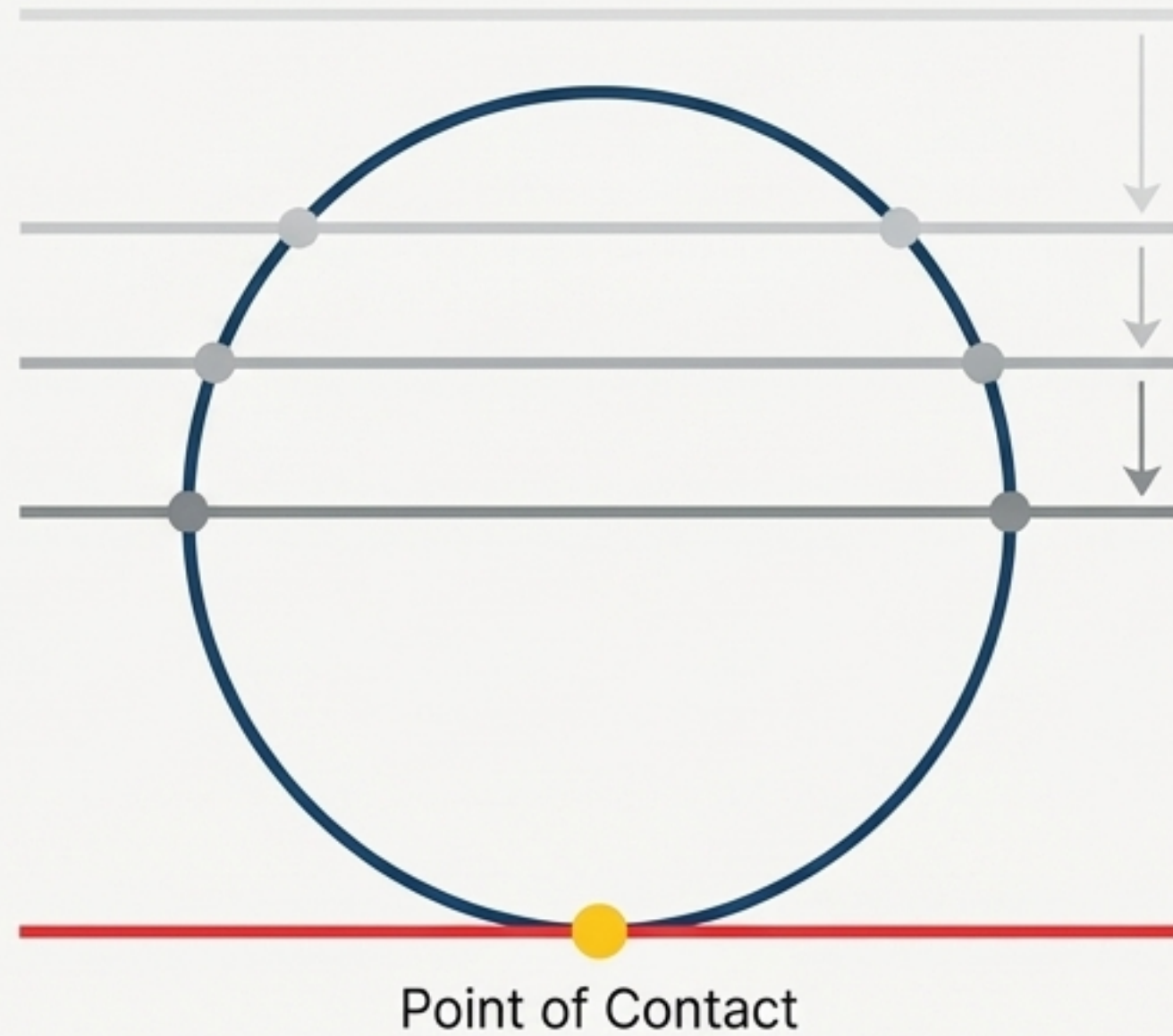


Secant
2 Points of Contact



Tangent
1 Point of Contact

The Moment of Contact

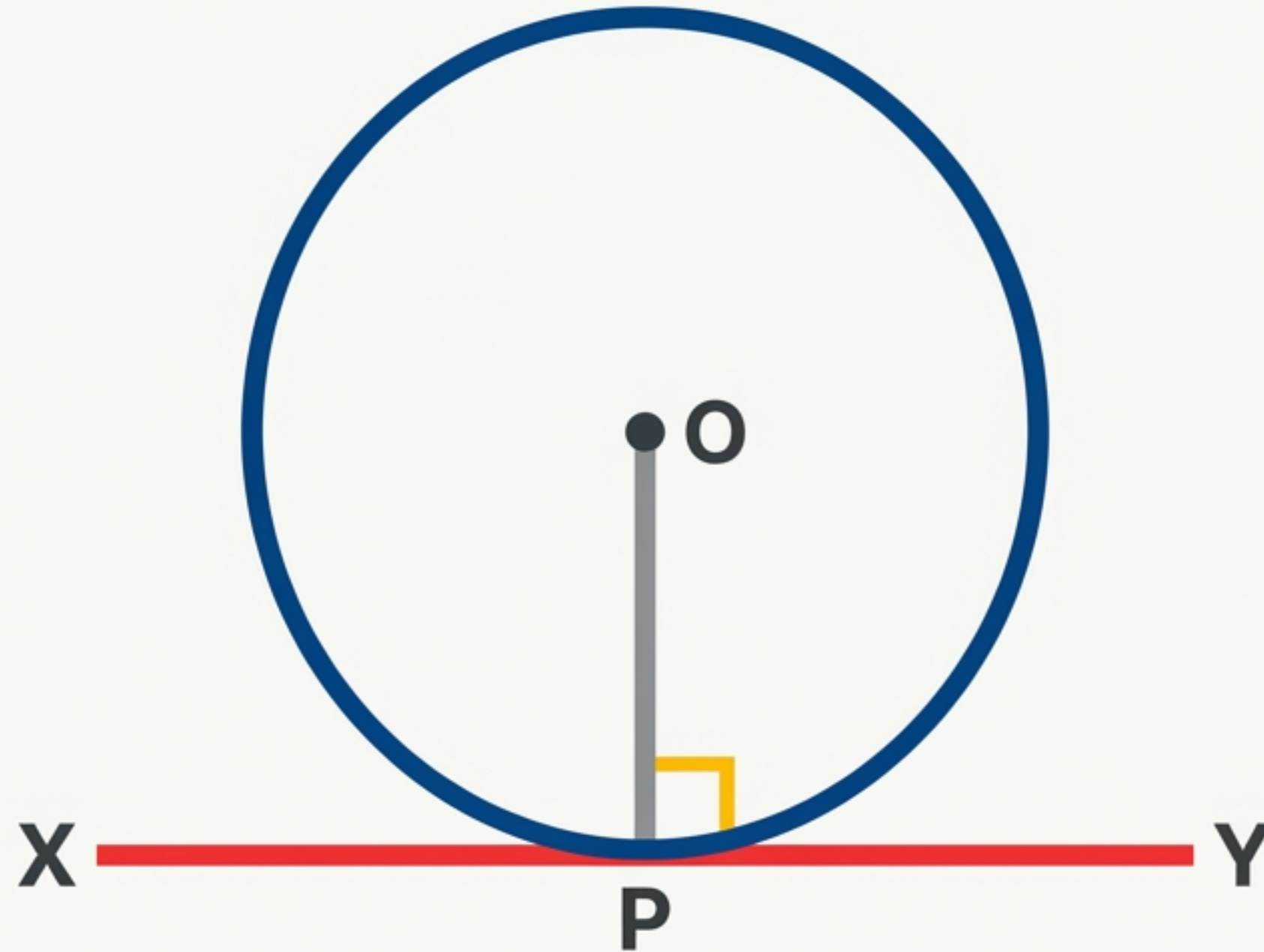


A tangent is simply a secant where the two end points of the chord coincide.



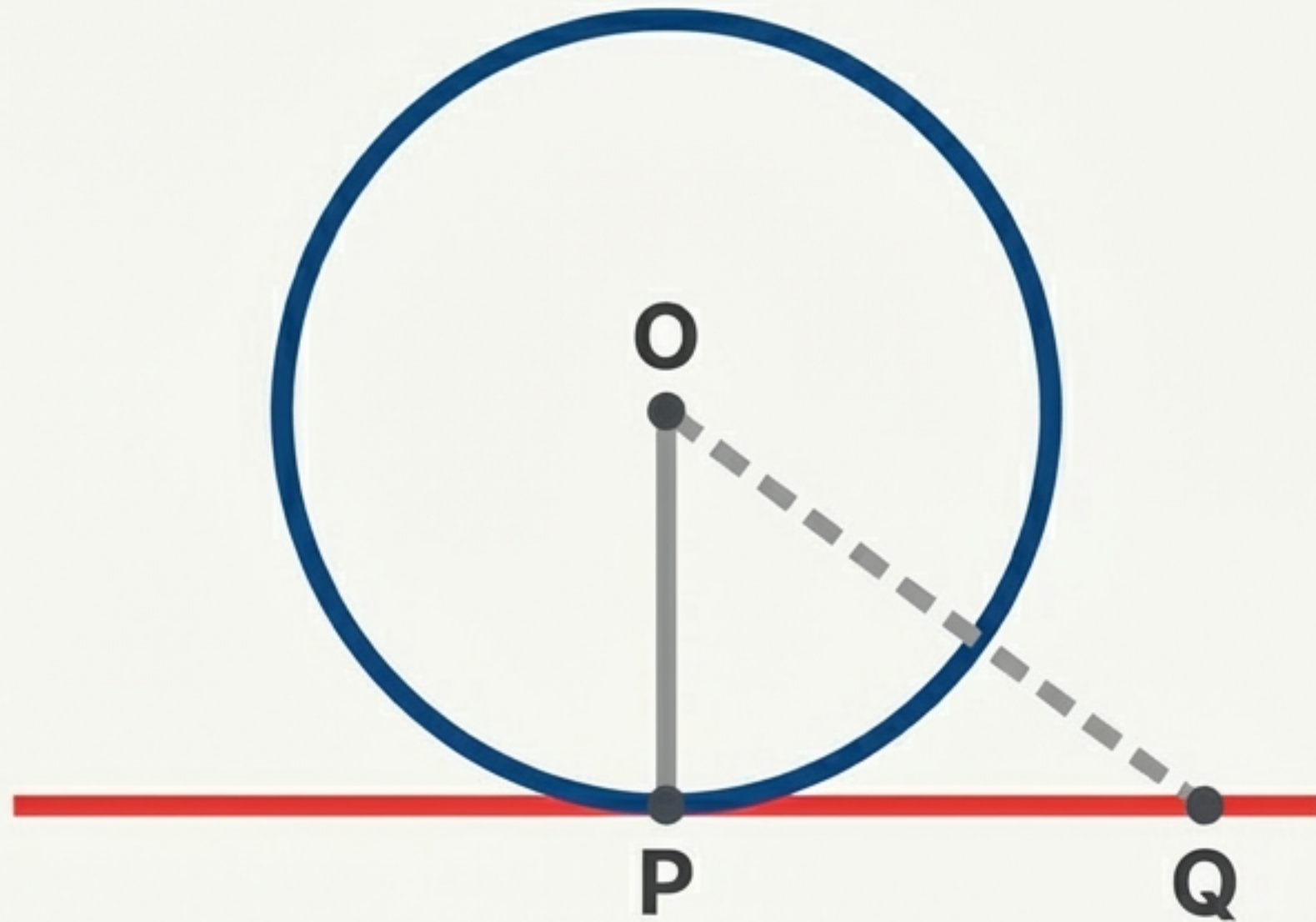
Real-world application: A bicycle wheel moves along a tangent line (the ground), and the spokes act as radii.

Theorem 10.1: The Perpendicular Rule



The tangent at any point of a circle is perpendicular to the radius through the point of contact.

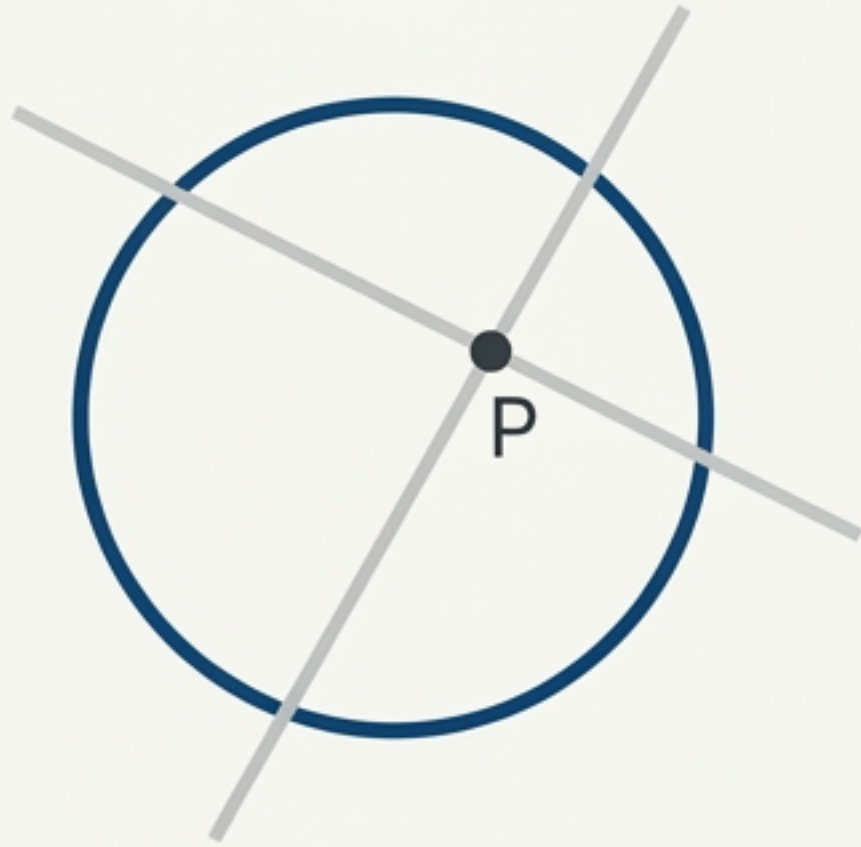
Proving the 90° Rule



- Any point Q on the tangent (other than P) must lie outside the circle.
- Therefore, distance OQ is always strictly longer than radius OP ($OQ > OP$).
- Because OP is the shortest possible path from the center to the line, it must be exactly perpendicular.

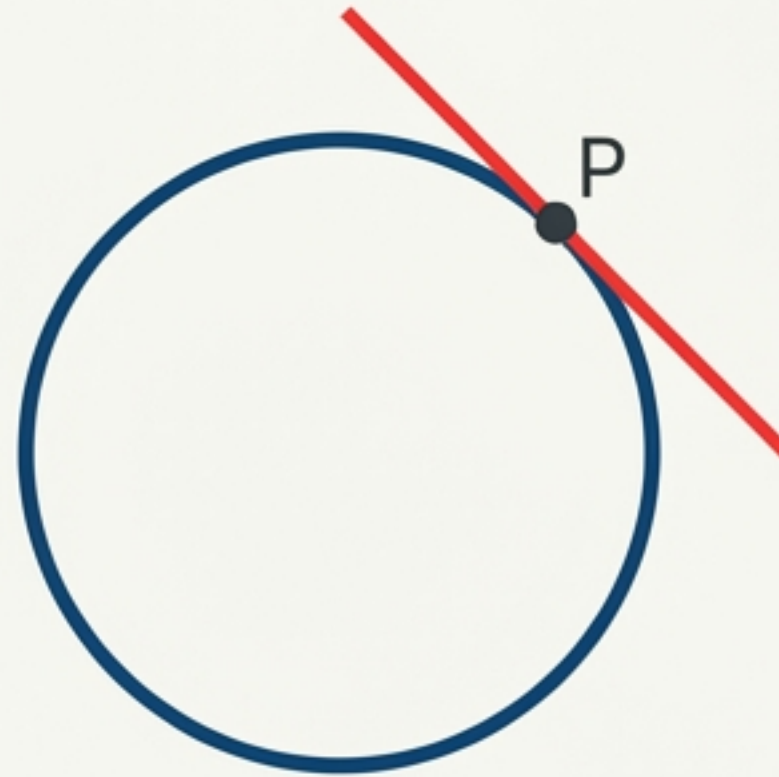
The Origin Point Test

Point Inside



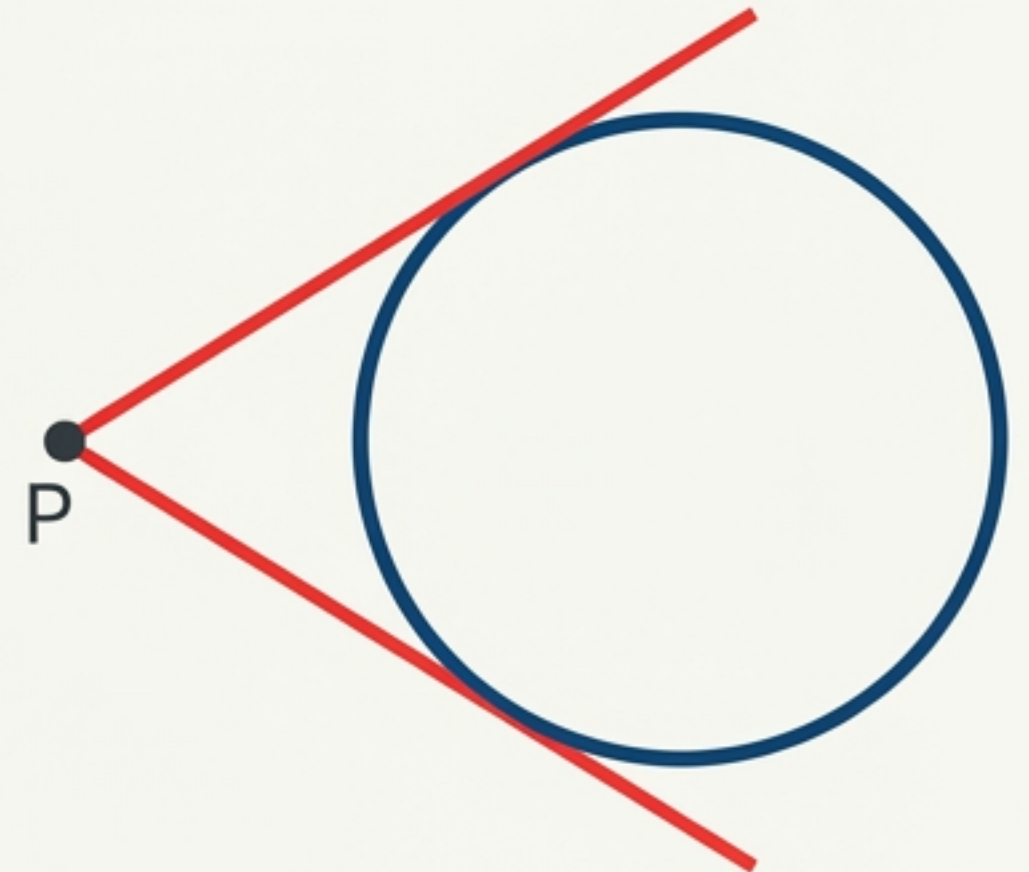
0 Tangents
(All lines are secants)

Point On Circle



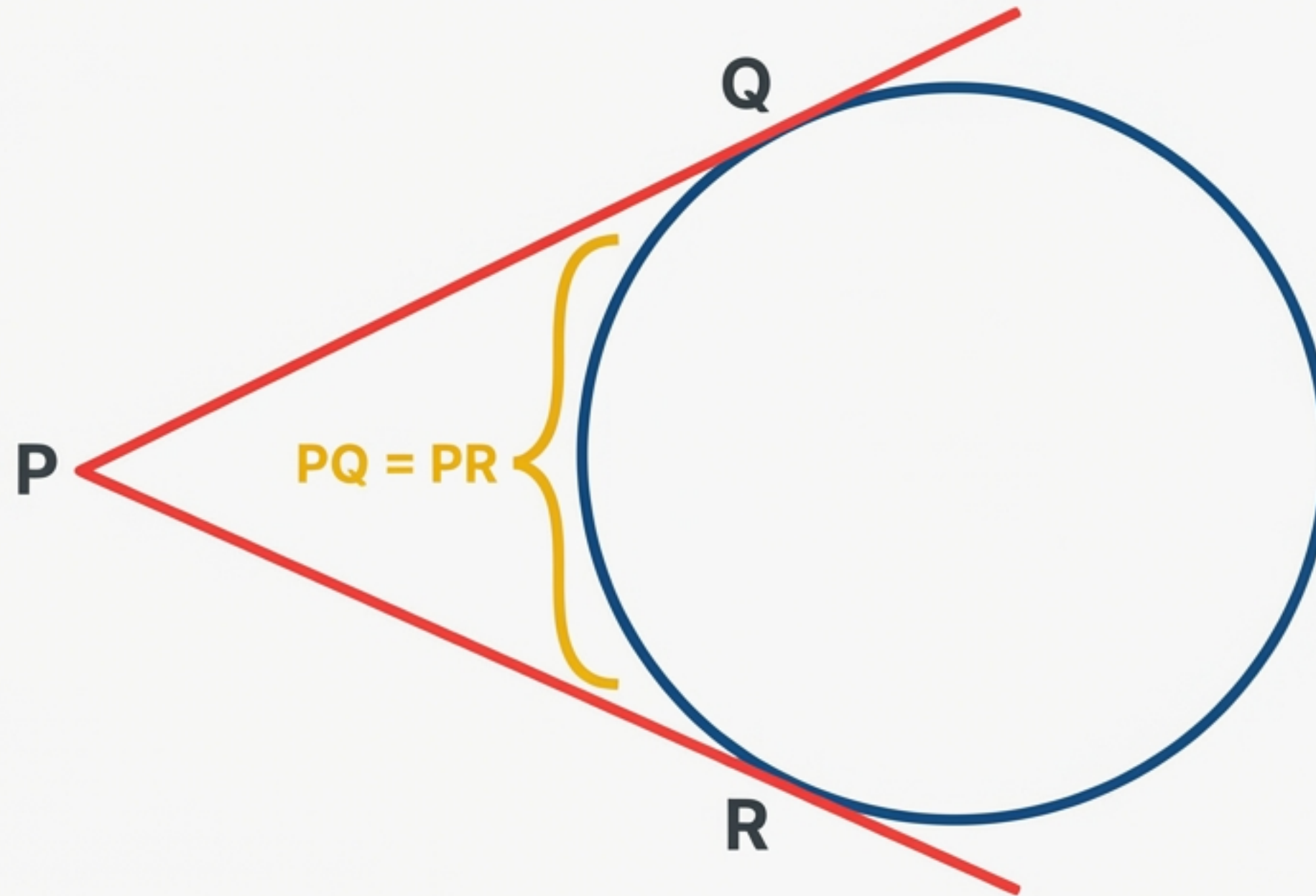
1 Tangent

Point Outside



Exactly 2 Tangents

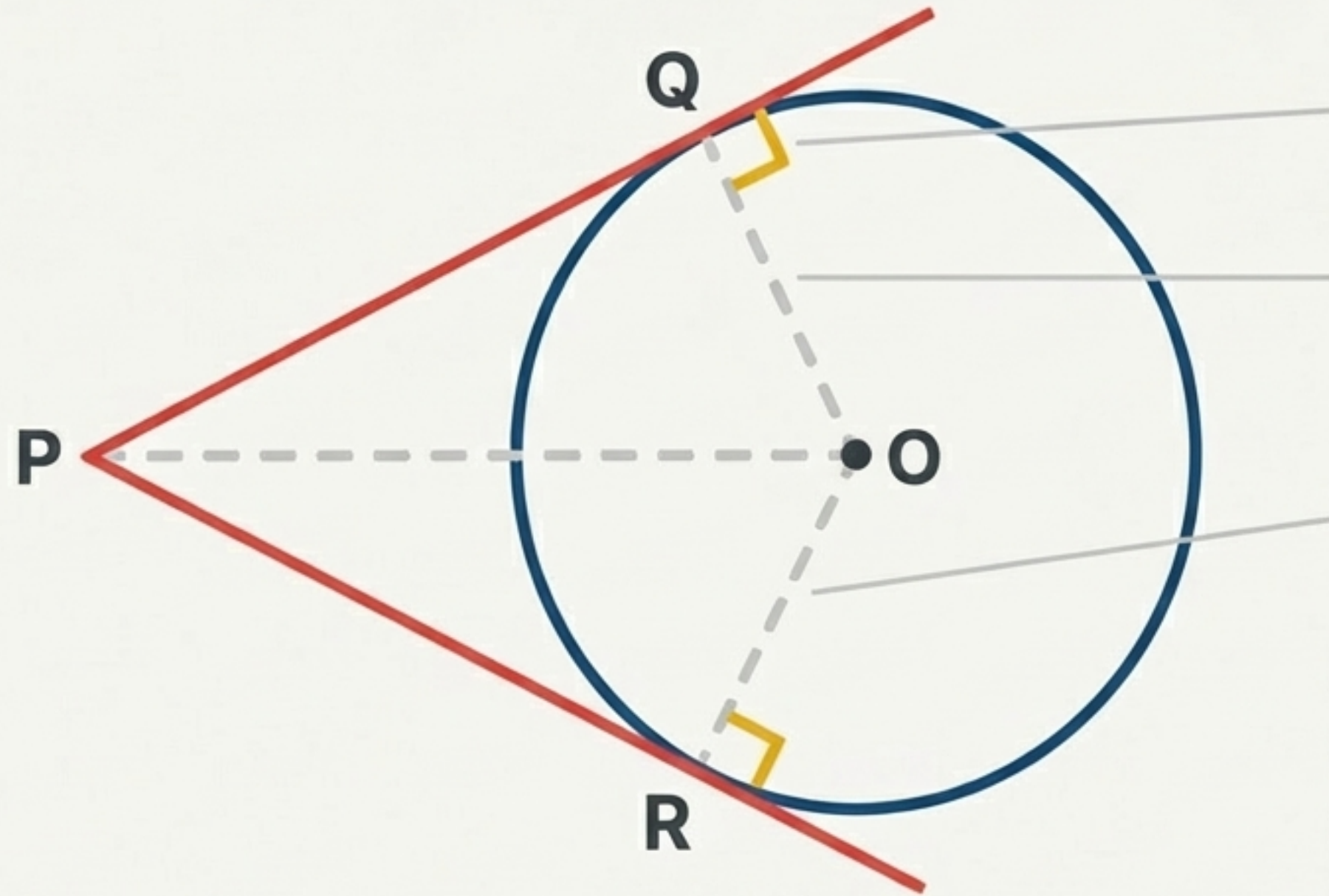
Theorem 10.2: The Equal Tangents Rule



The lengths of tangents drawn from an external point to a circle are exactly equal.

$$PQ = PR$$

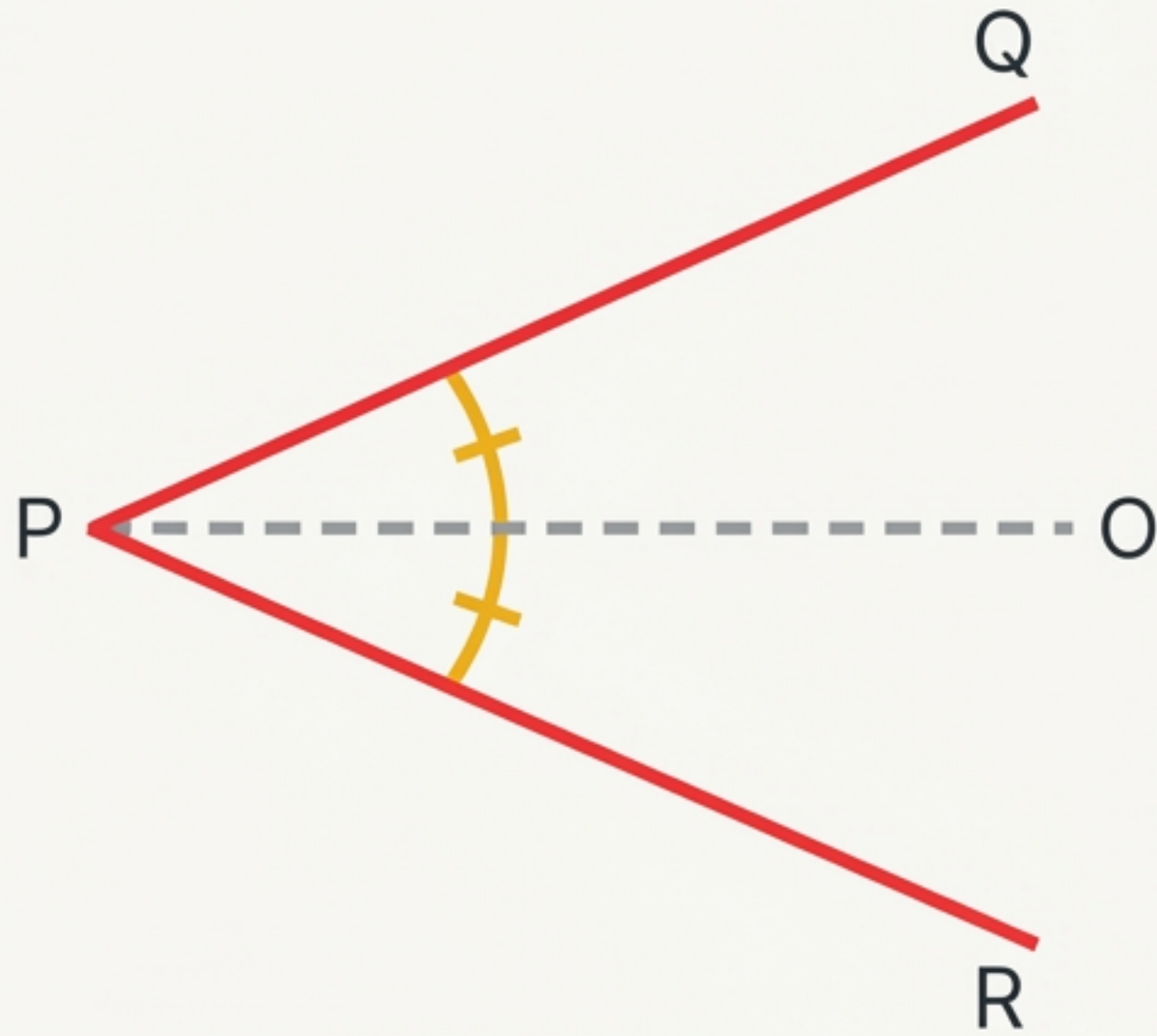
The Anatomy of Symmetry



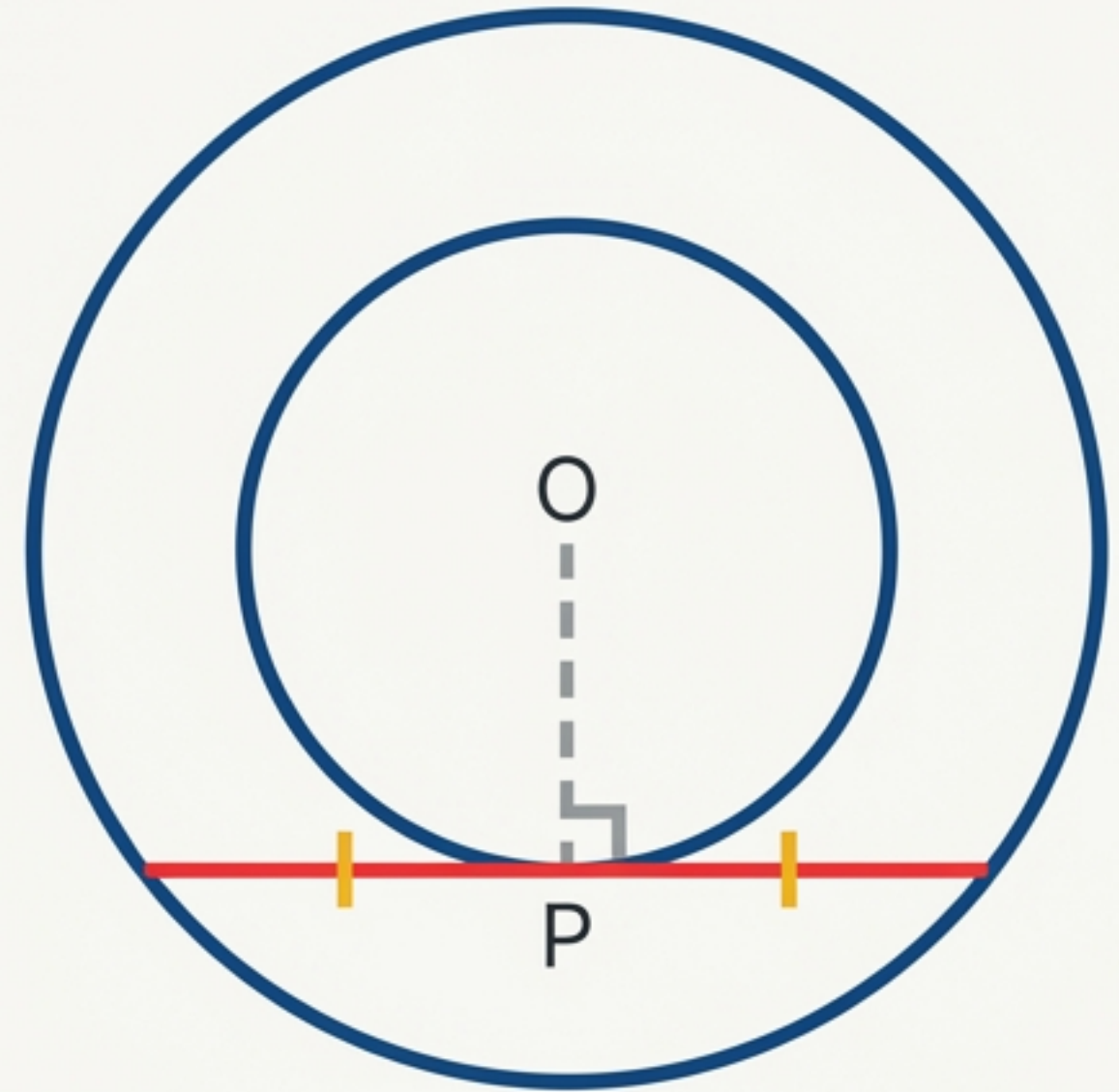
- Right Angles: $\angle OQP$ and $\angle ORP$ are 90°
- Hypotenuse: Both share the exact same hypotenuse (OP)
- Side: The radii are equal ($OQ = OR$)

By Right-Angle Hypotenuse Side (RHS) congruence, the top and bottom triangles are perfect mirror images. Therefore, PQ must equal PR.

Hidden Symmetries & Applications



The center always lies on the bisector of the angle between two intersecting tangents.



A chord of a larger circle tangent to a smaller concentric circle is perfectly bisected at the point of contact.

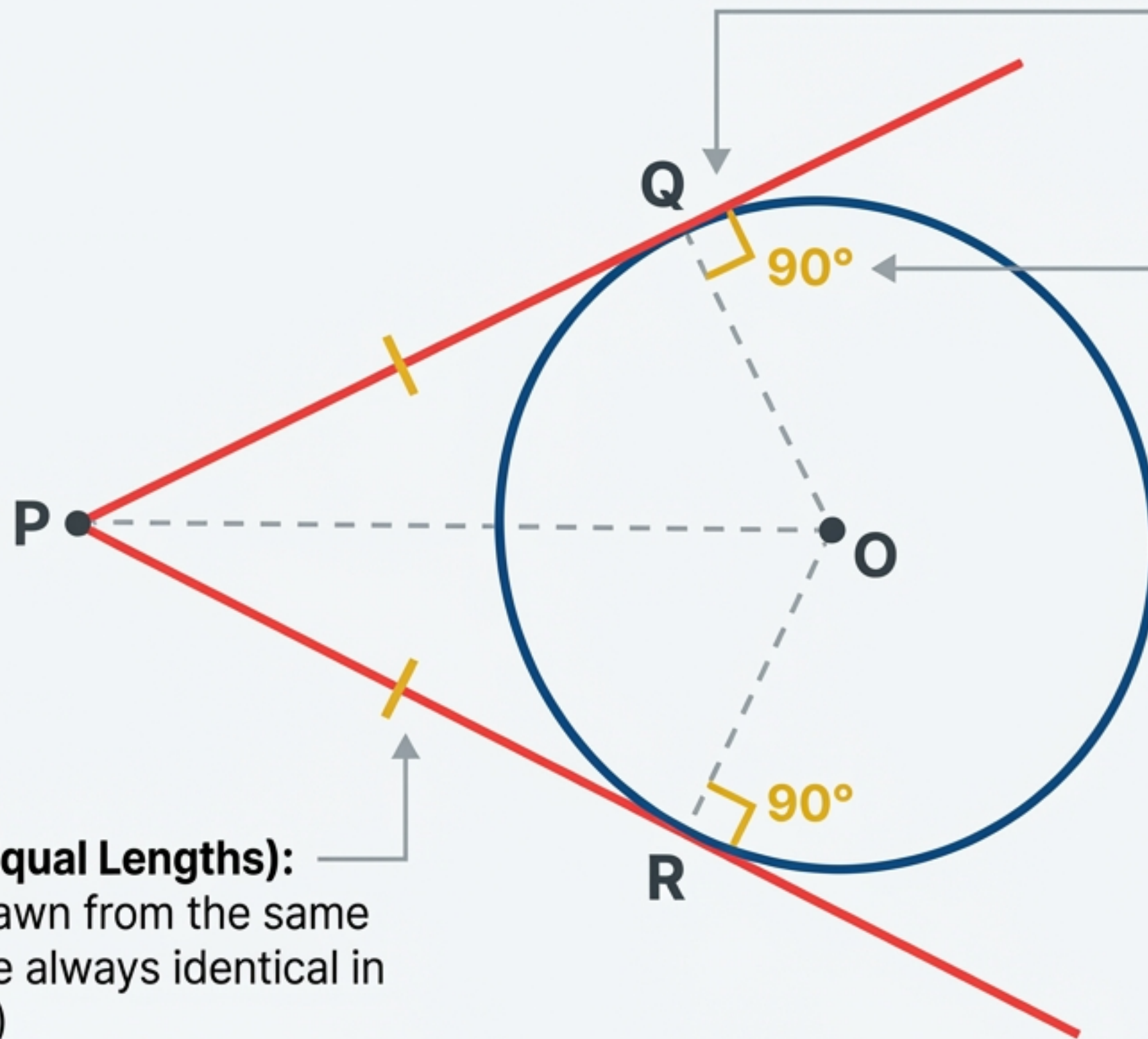
The Tangent Cheat Sheet

The Definition:

A line that touches the circle at only one common point (Point of Contact).

Theorem 10.1 (Perpendicularity):

The radius through the point of contact is always 90° to the tangent. ($OP \perp XY$)



Theorem 10.2 (Equal Lengths):

Two tangents drawn from the same external point are always identical in length. ($PQ = PR$)